Revenue Management under High-Variance Demand: Asymptotically Tight Fluid Approximations

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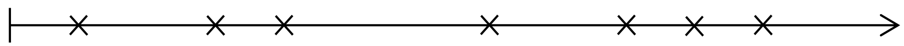
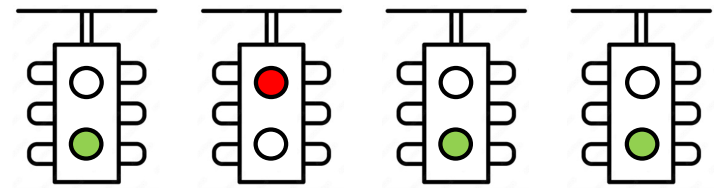


Traditional demand arrival models use a Poisson process

Variability in number of customer arrivals under a Poisson process is too small to be practically useful

How to build revenue management models with high-variance demand?

Revenue Management under High-Variance Demand



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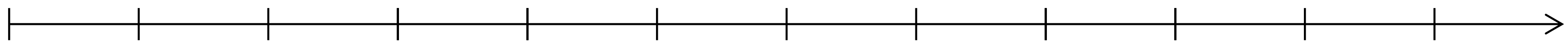
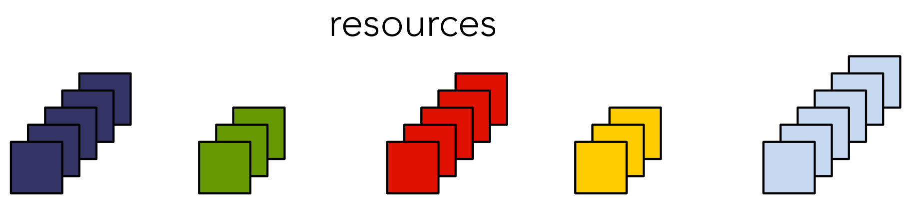
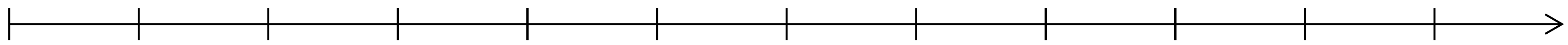
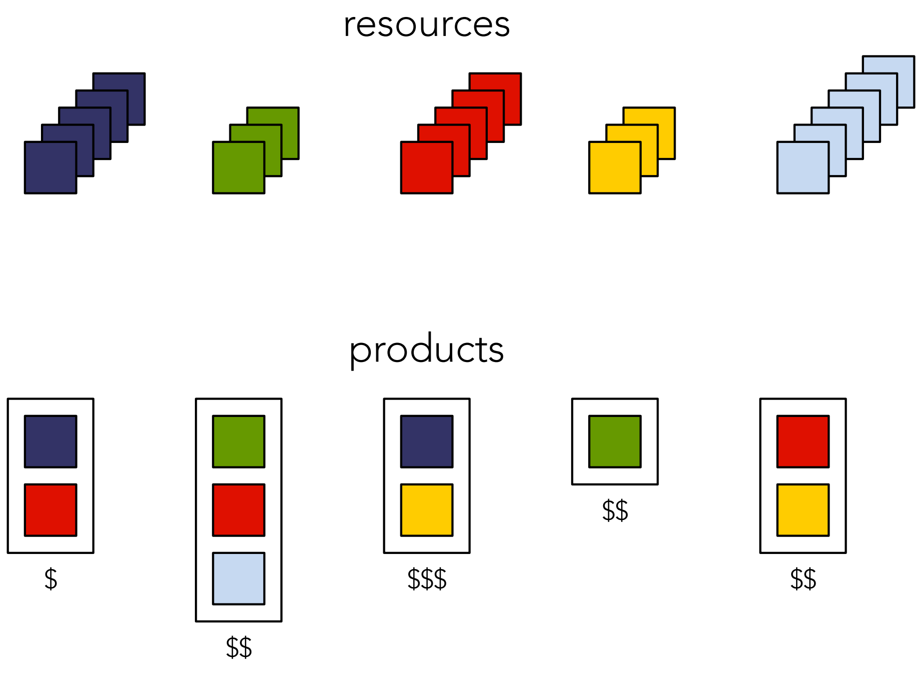
Revenue Management








resources
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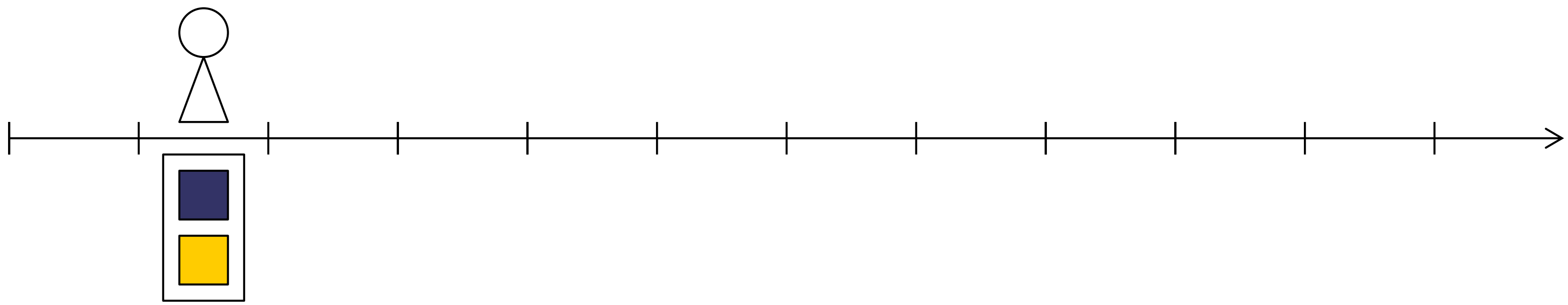
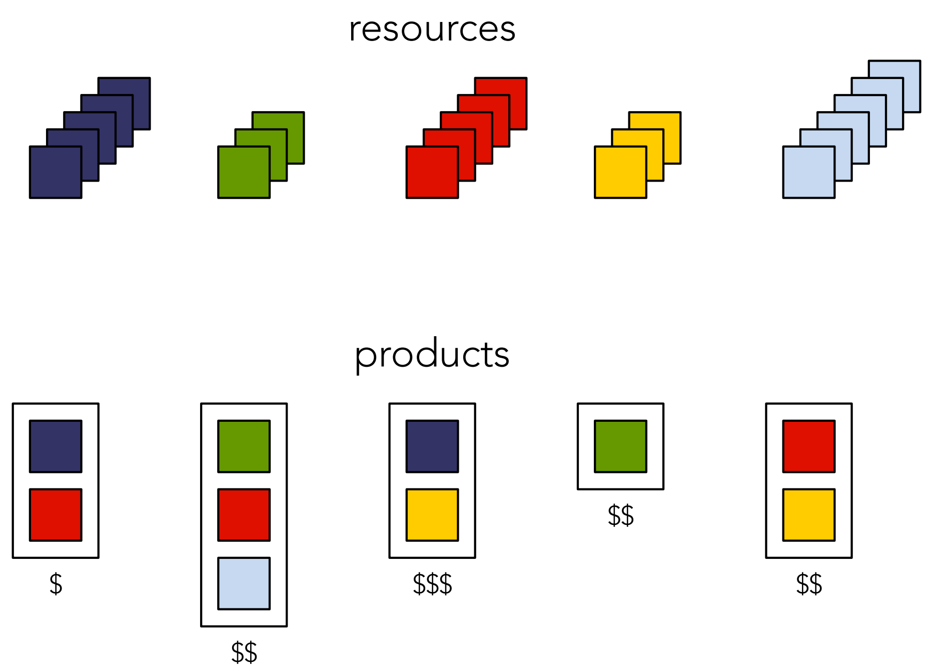
Revenue Management





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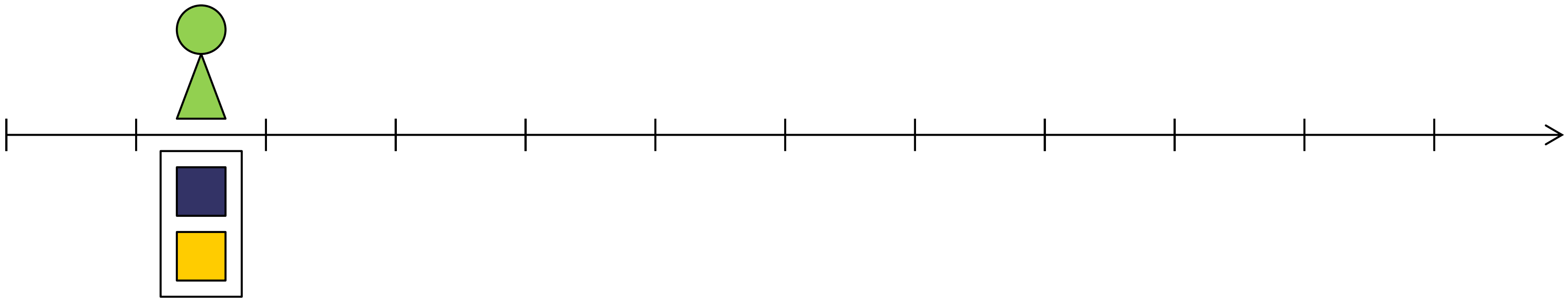

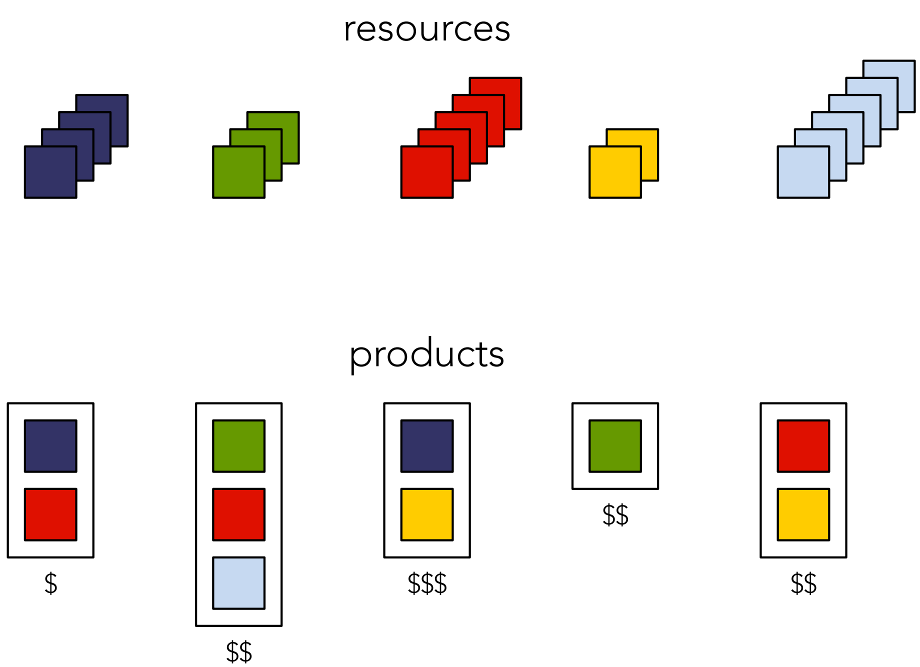

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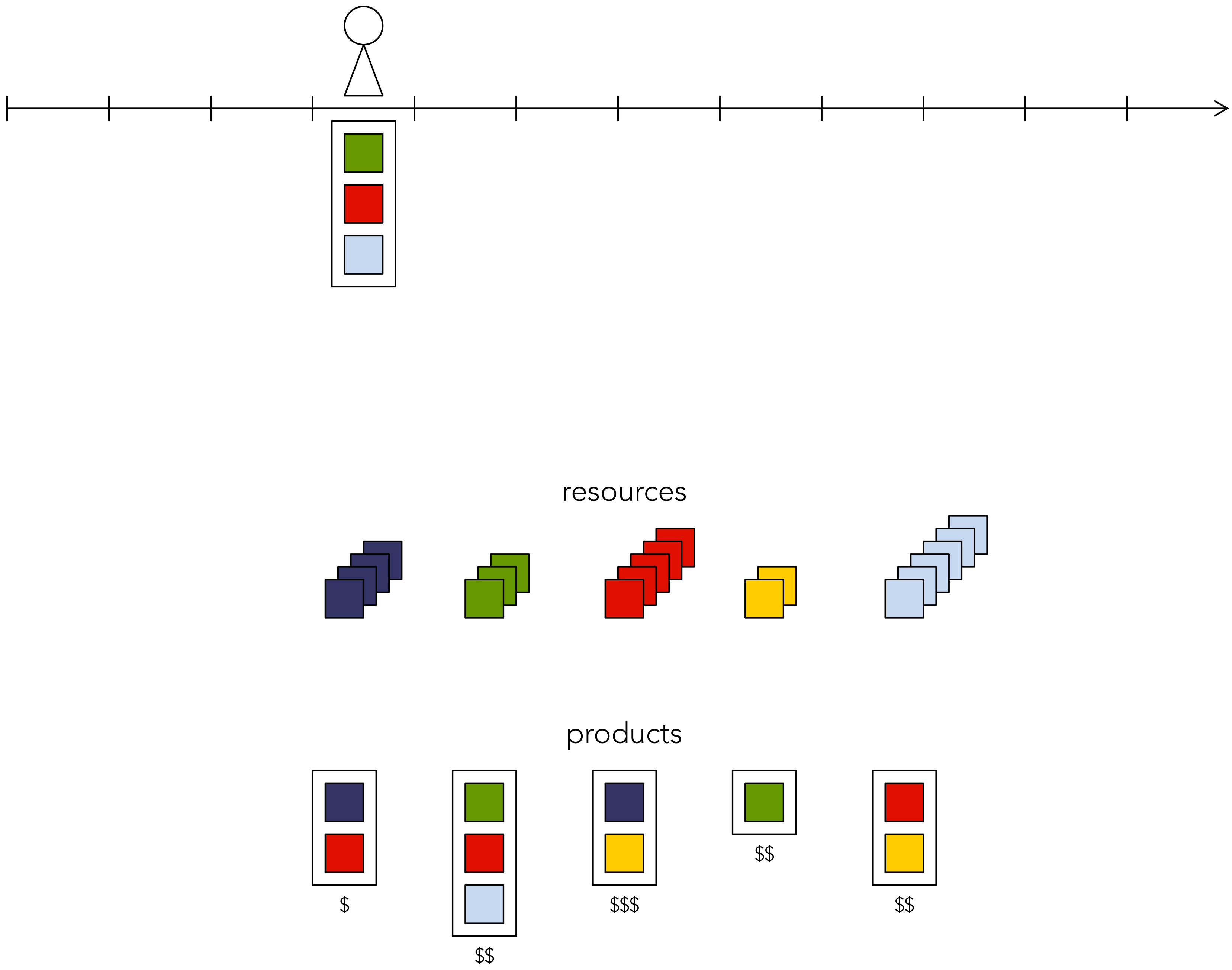

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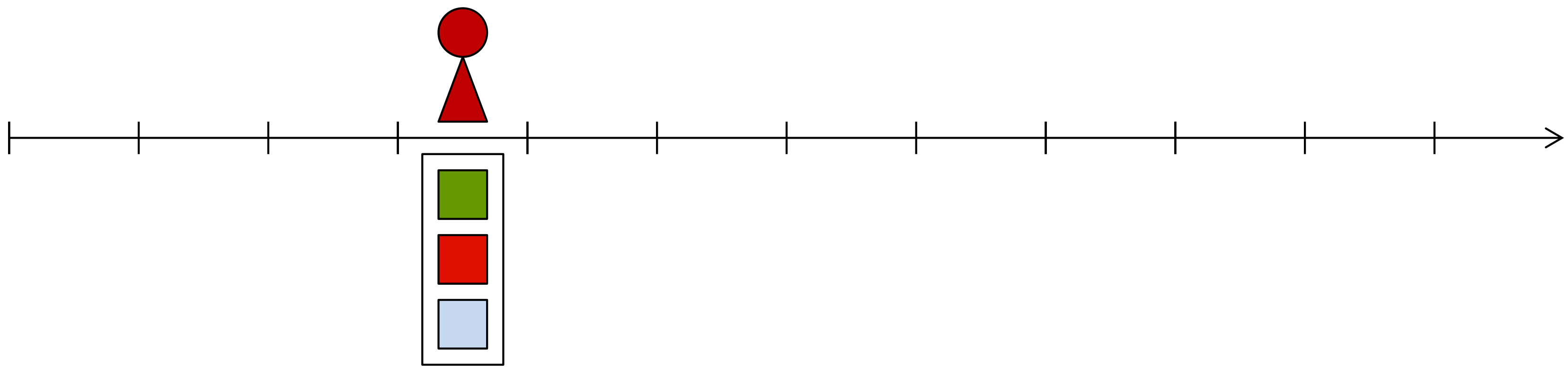

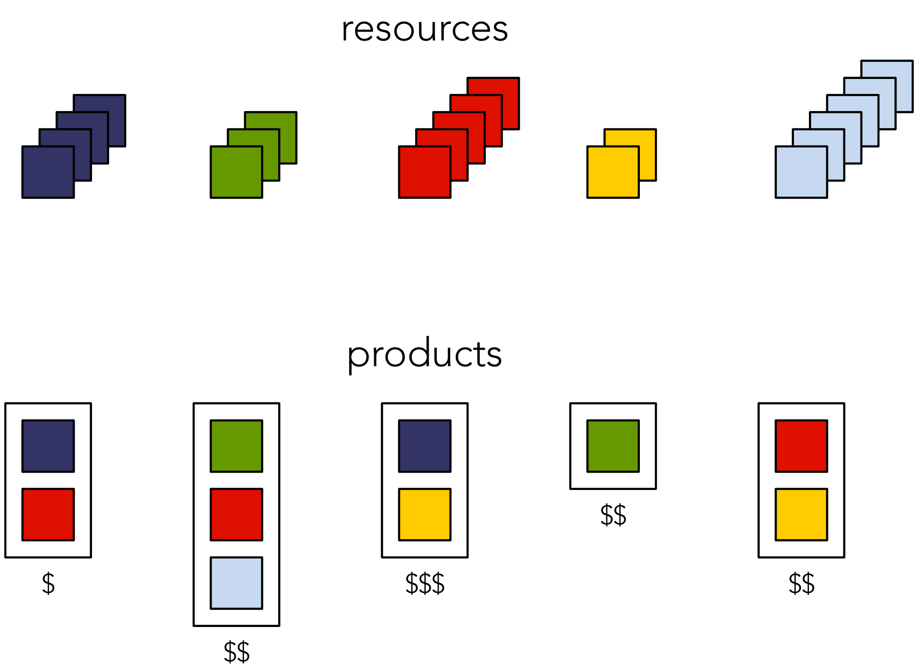

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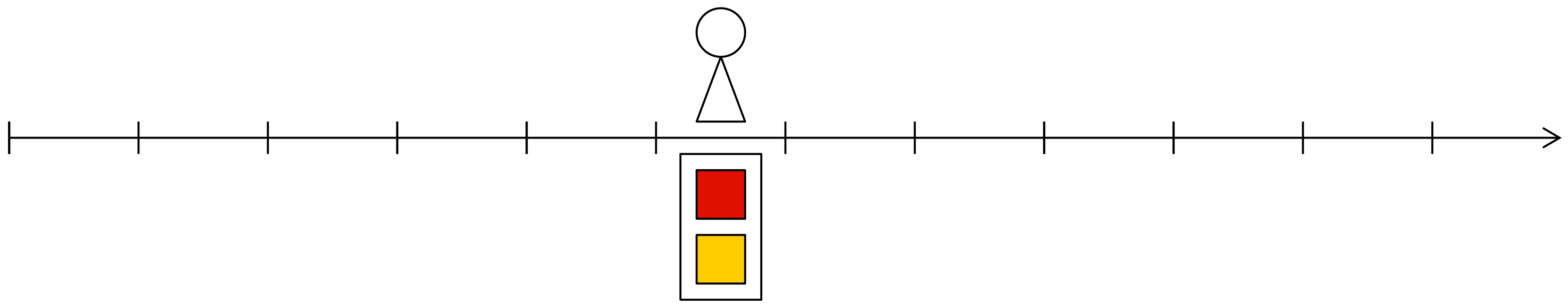

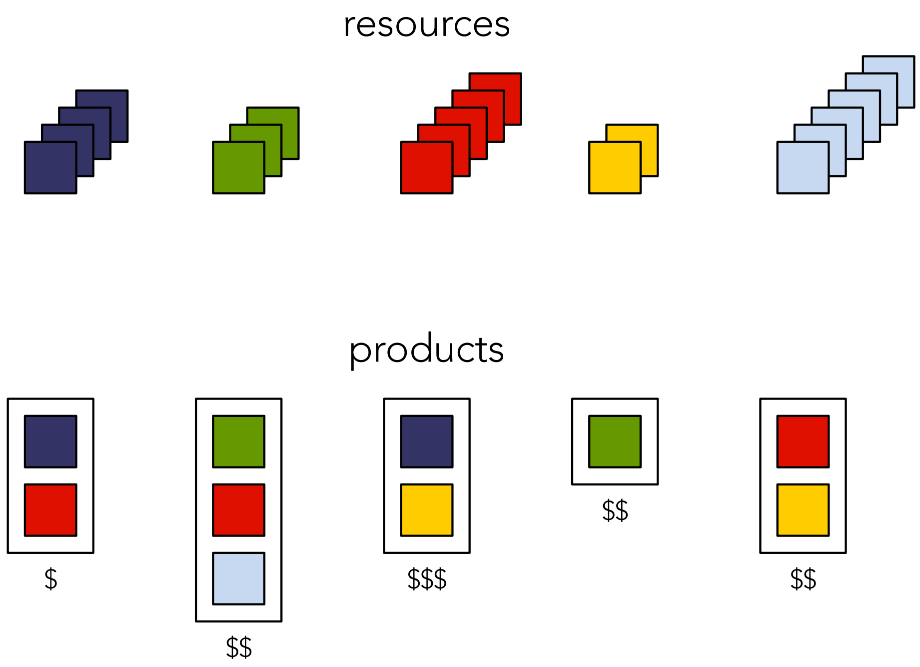

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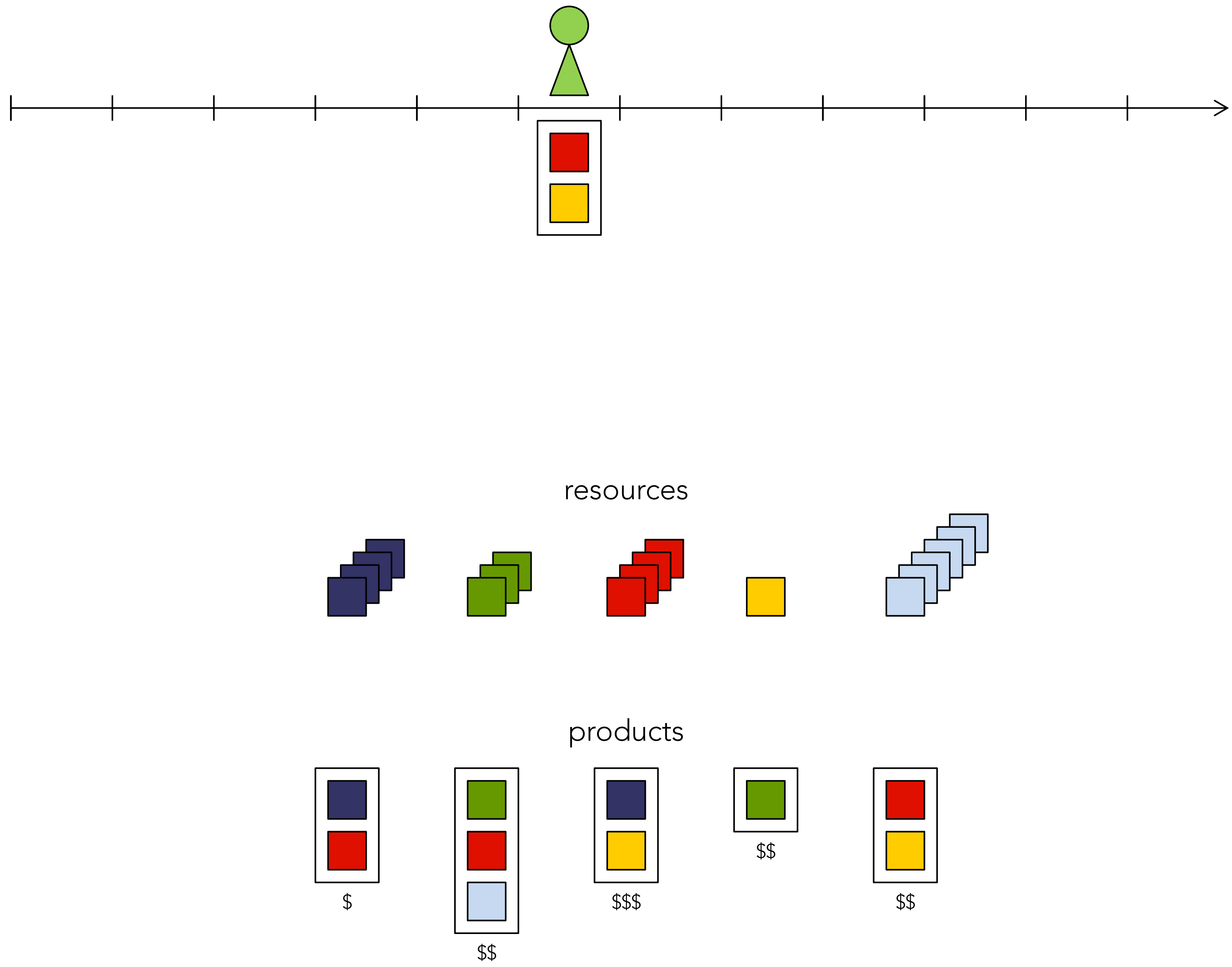
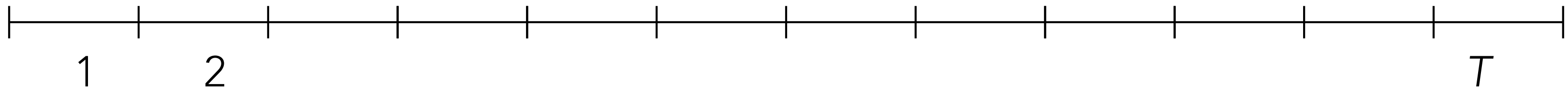


Revenue Management

Traditional Demand Model for Revenue Management



M	:	Set of resources
N	:	Set of products
c_i	:	Capacity of resource i
f_j	:	Revenue of product j
a_{ij}	:	1 iff product j uses resource i
T	:	Number of time periods in selling horizon
λ_{jt}	:	Request probability for product j at time period t

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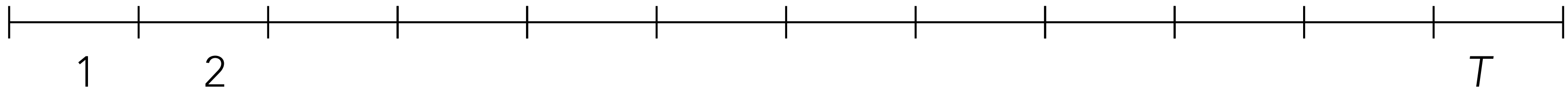

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$$V_t(x) = \max_{u \in \mathcal{F}(x)} \left\{ \sum_{j \in N} \lambda_{jt} \left\{ f_j \, u_j + V_{t+1}\Big(x - \sum_{i \in M} e_i \, a_{ij} \, u_j \Big) \right\} \right\}$$

$$\mathcal{F}(x) = \left\{ u \in \{0,1\}^N : a_{ij} \, u_j \leq x_i \;\; \forall \, i \in M, \; j \in N \right\}$$

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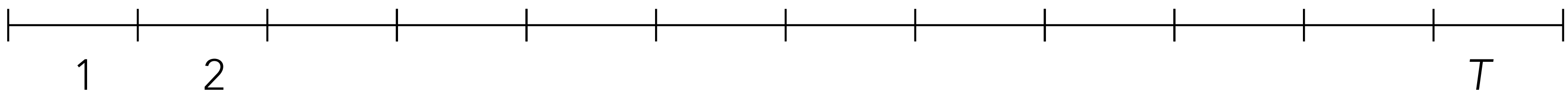

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$$\text{OPT} = V_1(c)$$

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Fluid Approximation under Traditional Demand Model

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$\text{FLD} \geq \text{OPT}$

Approximate Policy

Accept a request for product j at time period t with probability $\overline{y}_{jt}/\lambda_{jt}$ as long as there is enough capacity to accept the request

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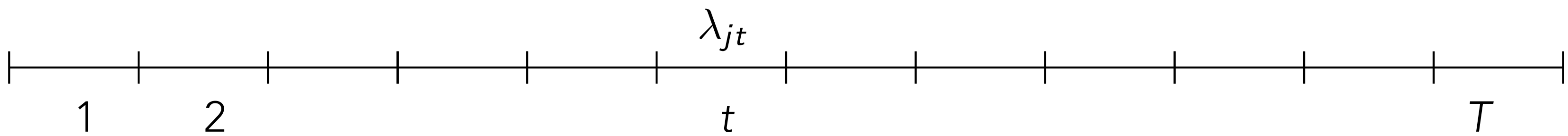
Accept a request for product j at time period t with probability $\bar{y}_{jt}/\lambda_{jt}$ as long as there is enough capacity to accept the request

$$\text{APX} \geq \max \left\{ \frac{1}{4L}, 1 - (1 + L) \sqrt{\frac{2 \log C}{C}} \right\} \text{OPT}$$

$C = \min_{i \in M} c_i, \;\; L = \max_{j \in N} \sum_{i \in M} a_{ij}$

Traditional Demand Model for Revenue Management



Expected demand
for product j

$$\sum_{t \in T} \lambda_{jt}$$

Standard deviation of demand
for product j

$$\sqrt{\sum_{t \in T} \lambda_{jt}(1 - \lambda_{jt})} \;\leq\; \sqrt{\sum_{t \in T} \lambda_{jt}}$$

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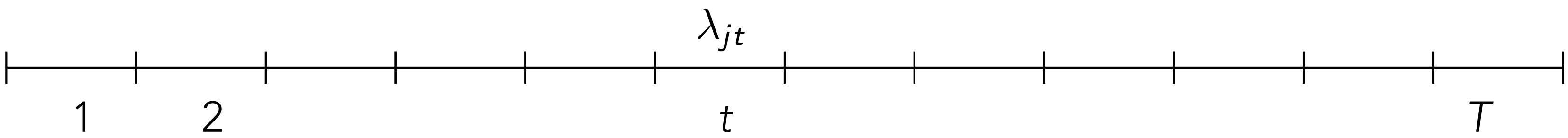

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Large demand volume and large demand variability cannot coexist in traditional demand model!

Traditional Demand Model for Revenue Management

How to build network revenue management models that accommodate large demand variability?

How to build fluid approximations with sound footing when demand has large variability?

Do fluid approximations work because demand variability vanishes with large demand volume?

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$$\text{FLD} \geq \text{OPT} \qquad \frac{\text{APX}}{\text{OPT}} \xrightarrow{c\to\infty} 1 \qquad \frac{\text{OPT}}{\text{FLD}} \xrightarrow{c\to\infty} 1$$

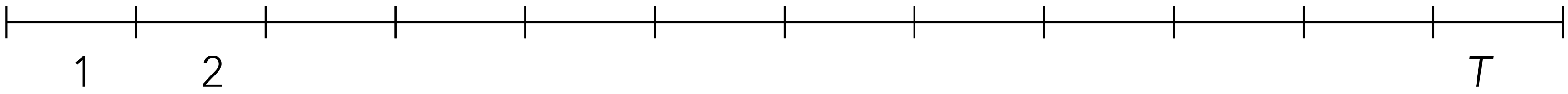
Do fluid approximations work because demand variability vanishes with large demand volume?

Outline

High-variance demand

Calendar-aware demand

High-Variance Demand Model



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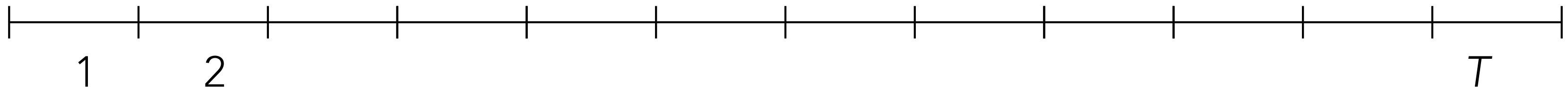
$$\theta_t = \mathbb{P}\Big\{D \geq t+1 \,|\, D \geq t\Big\}$$

High-Variance Demand Model



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Single resource with capacity C, single product with revenue of 1,
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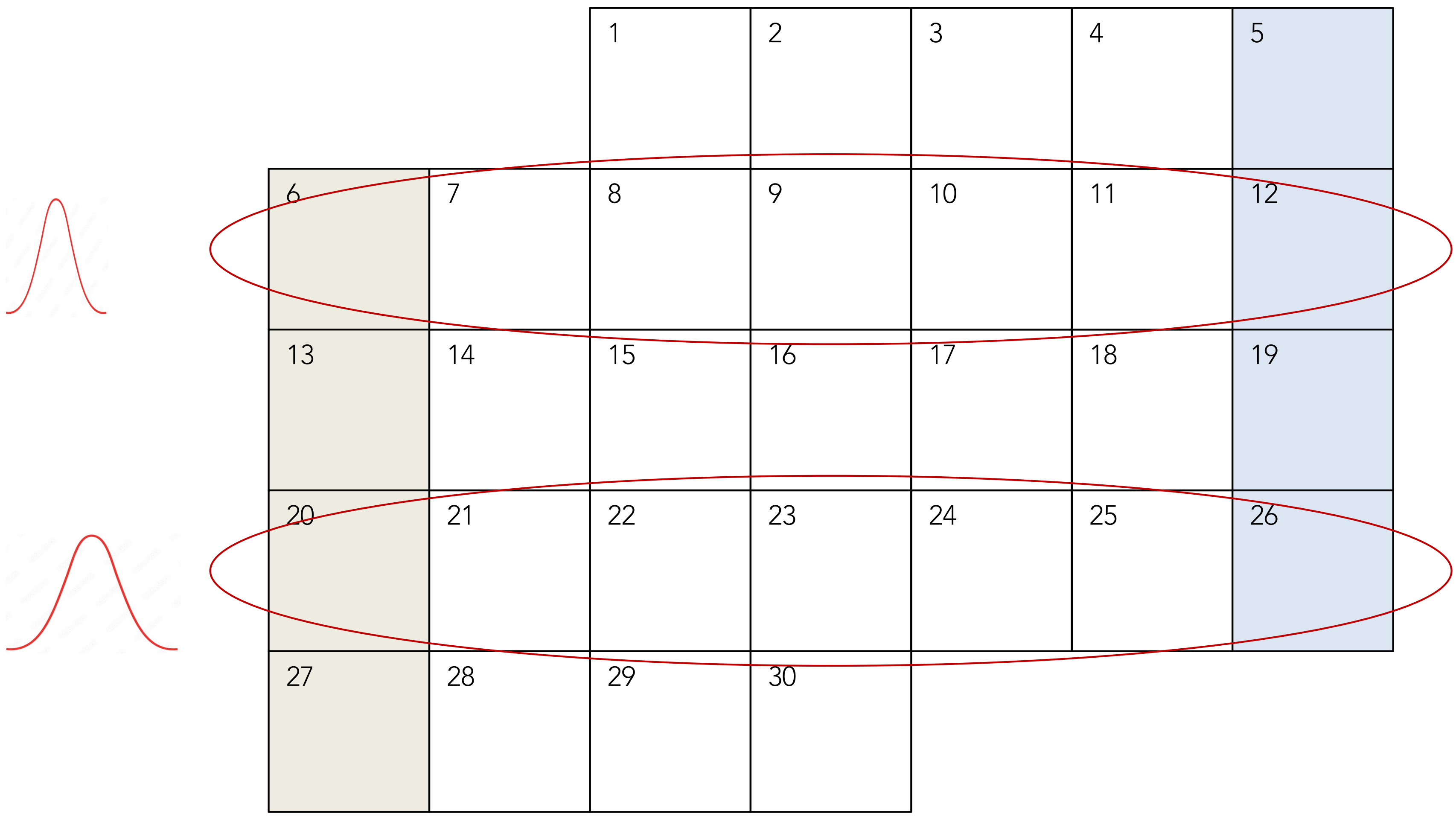
Calendar-Aware Demand Model



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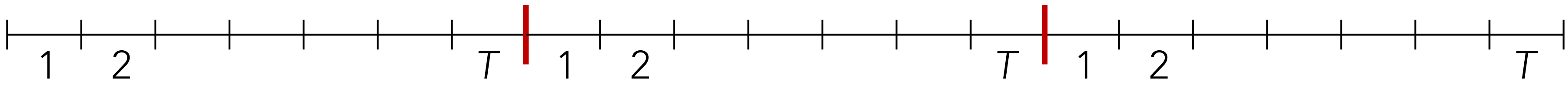

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$$\theta^k_t = \mathbb{P}\left\{ D^k \geq t+1 \mid D^k \geq t \right\}$$

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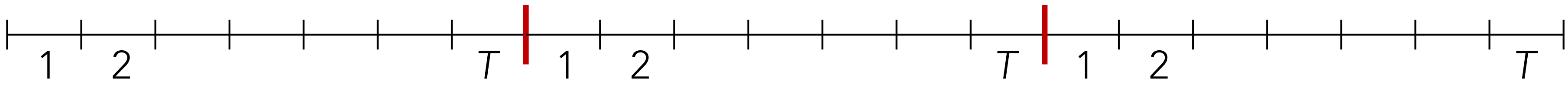

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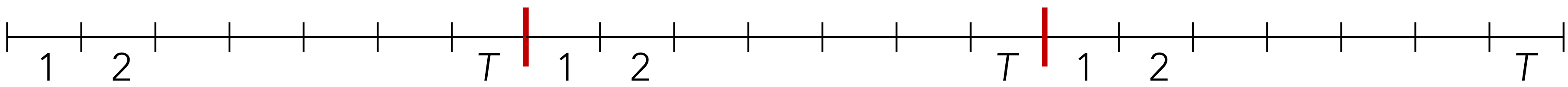

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$$V_t^k(x) = \max_{u \in \mathcal{F}(x)} \left\{ \sum_{j \in N} \lambda_{jt}^k \left\{ f_j\, u_j + \theta_t^k\, V_{t+1}^k \left(x - \sum_{i \in M} e_i\, a_{ij}\, u_j \right) + (1 - \theta_t^k)\, V_1^{k+1} \left(x - \sum_{i \in M} e_i\, a_{ij}\, u_j \right) \right\} \right\}$$

D^k is sub-Gaussian with variance proxy σ^2

$$\theta_t^k = \mathbb{P}\left\{ D^k \geq t+1 \,|\, D^k \geq t \right\}$$

Fluid Approximation under Calendar-Aware Demand

y^k_{jt} : Probability of accepting a request for product j from customer t in stage k

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$$K = 2, \quad T = 2, \quad C = 3, \quad L = 1, \quad \frac{\text{FLD}}{\text{OPT}} = \frac{10}{11}$$

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$$\text{FLD} \geq \text{OPT}$$

Approximate Policy

Accept a request for product j from customer t in stage k with probability $\bar{y}_{jt}^k/\lambda_{jt}^k$ as long as there is enough capacity to accept the request

$$\text{APX} \geq \max\left\{\frac{1}{4L},\, 1 - (4+L)\frac{\sqrt{(C + \sigma^2\,(K-1))\log C}}{C}\right\}\text{OPT}$$

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Single resource with capacity C, single product with revenue 1, selling horizon with K stages, demands in different stages are iid, all customers request the product

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Outlook

Calendar-aware and dependent demand
Pricing and assortment decisions
Stronger performance guarantees under calendar-aware demand

Revenue Management with Calendar-Aware and Dependent Demands:
Asymptotically Tight Fluid Approximations
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