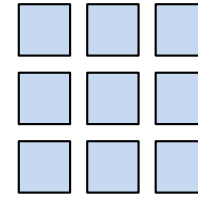
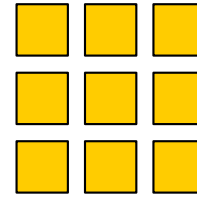
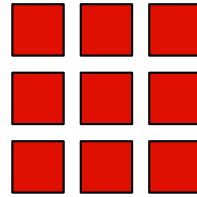
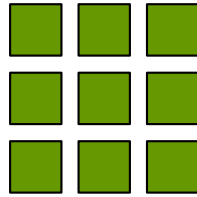
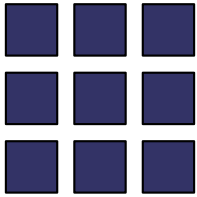


# Coordinated Inventory Stocking and Assortment Customization

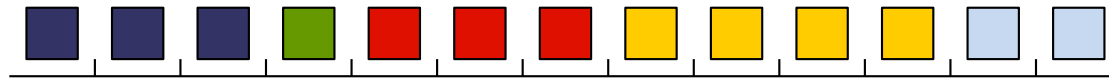
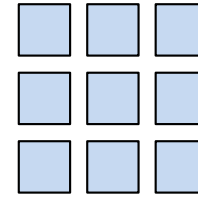
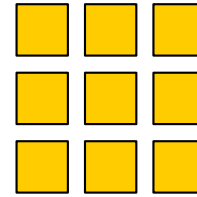
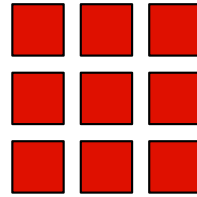
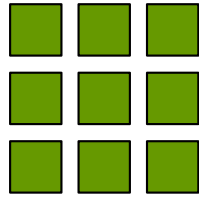
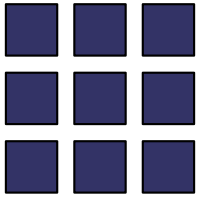
Huseyin Topaloglu  
Cornell Tech

Yicheng Bai, Omar El Housni, Paat Rusmevichientong

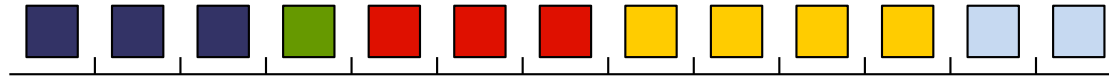
# Joint Inventory Stocking and Assortment Customization



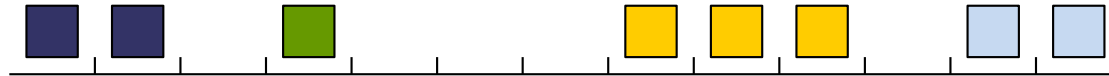
# Joint Inventory Stocking and Assortment Customization



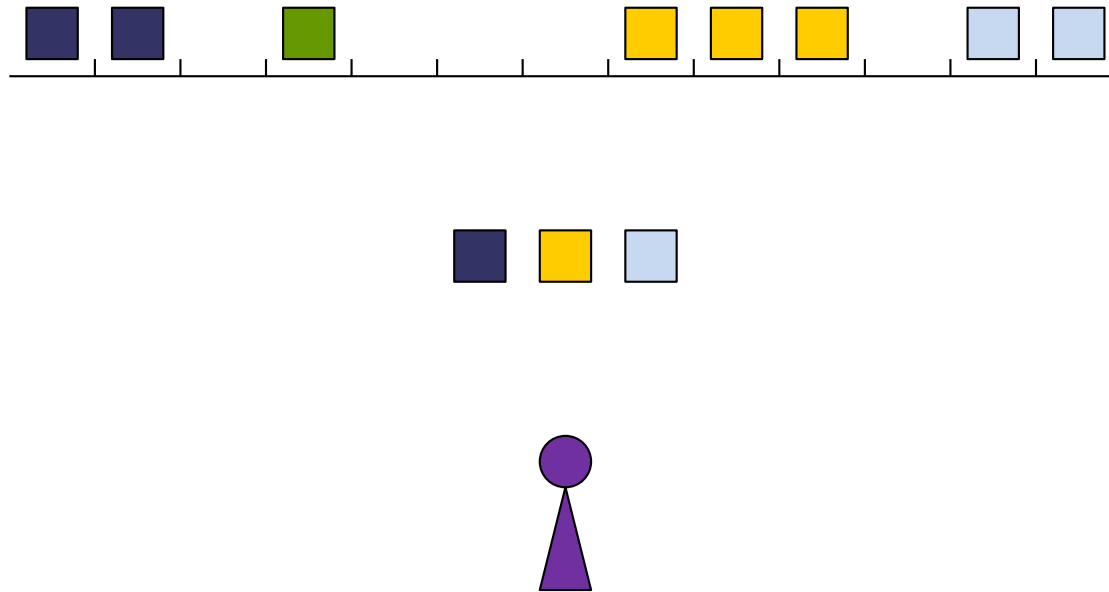
# Joint Inventory Stocking and Assortment Customization



# Joint Inventory Stocking and Assortment Customization



# Joint Inventory Stocking and Assortment Customization



# Joint Inventory Stocking and Assortment Customization

amazon **Deliver to: New York 10011** Whole Foods Market **yoghurt** Hello, Name! Account & Lists Returns & Orders Cart

WHOLE FOODS MARKET Past Purchases Deals Alexa lists Recipes Food Beverages Household Personal Care, Health & Beauty Body Care & Beauty In-Store

1-24 of 106 results for "yoghurt" Sort by: Featured

**Department**  
Any Department  
Whole Foods Market  
**Grocery & Gourmet Food**  
Baby Foods  
Beverages  
Breakfast Foods  
Dairy, Cheese & Eggs  
Frozen  
Pantry Staples  
Produce  
Snack Foods

**Avg. Customer Review**  
★★★★☆ & up  
★★★★☆ & up  
★★★★☆ & up  
★★★★☆ & up

**Featured Brands**  
Stonyfield Organic  
Chobani  
Noosa  
Fage  
siggi's  
Kite Hill  
Brown Cow

**Price**  
Under \$10  
\$ Min \$ Max Go

**Specialty**  
Gluten-Free  
Kosher  
Organic  
Vegan  
USDA Organic  
Vegetarian  
GMO-Free

**Calories Per Serving**  
Up to 40 Calories  
40 to 100 Calories  
100 to 200 Calories  
200 to 300 Calories

**Fat Calories Per Serving**  
0 Calories  
40-100 Calories

**Nutrition Facts**  
Fat Free (<0.5g)  
Low Fat (<5g)  
Free of Saturated Fat (<0.5g)

**Showing results in Grocery & Gourmet Food**  
[Show instead results in all Whole Foods Market](#)

Product	Price	Rating	Count
Noosa Yoghurt, Vanilla Bean, 24 oz. Whole Milk Yogurt, Grade-A Pasteurized, Gluten Free, Probiotic, Made With the Finest Quality...	\$14.99 (\$0.21/Ounce)	★★★★☆	416
365 by WFM, Yogurt Plain Organic, 32 Ounce	\$3.99 (\$0.11/Ounce)	★★★★☆	4,849
Stonyfield Organic Whole Milk Probiotic Yogurt, Plain, 32 oz. -- Immunity & Digestive Health	\$14.49 (\$0.15/Ounce)	★★★★☆	5,442
Noosa Yoghurt, Vanilla, 16 oz	\$15.49 (\$0.34/Ounce)	★★★★☆	780
noosa Yoghurt, Vanilla Bean, 8 oz, Whole Milk Yogurt, Grade-A, Pasteurized, Gluten Free, Probiotic, Made With the Finest Quality...	\$1.49 (\$0.19/Ounce) \$0.99 33% off! Includes Prime savings	★★★★☆	876
noosa Yoghurt, Coconut, 8 oz, Whole Milk Yogurt, Grade-A, Pasteurized, Gluten Free, Probiotic, Made With All Natural Premium Quality Ingredients	\$1.49 (\$0.19/Ounce) \$0.99 35% off! Includes Prime savings	★★★★☆	1,647
Noosa Yoghurt, Blueberry, 16 oz	\$15.49 (\$0.34/Ounce)	★★★★☆	607
Stonyfield Organic Lowfat Yogurt, Plain, 32 oz. -- 7g of Protein, Multiserving Yogurt Snack & Cooking Substitute	\$14.29 (\$0.13/Ounce)	★★★★☆	1,144

# Joint Inventory Stocking and Assortment Customization





# Joint Inventory Stocking and Assortment Customization



# Joint Inventory Stocking and Assortment Customization



# Joint Inventory Stocking and Assortment Customization



## Optimization Model

$M$  : Set of customer types

$N$  : Set of products

$K$  : Storage size

$r_i$  : Revenue of product  $i$

$T$  : Number of time periods in selling horizon

$\lambda_{jt}$  : Probability that a customer of type  $j$  arrives at time period  $t$

$\phi_{ij}(S)$  : Choice probability of product  $i$  by a customer of type  $j$  from assortment  $S$

$x_i$  : Remaining inventory of product  $i$

- $M$  : Set of customer types
- $N$  : Set of products
- $K$  : Storage size
- $r_i$  : Revenue of product  $i$
- $T$  : Number of time periods in selling horizon
- $\lambda_{jt}$  : Probability that a customer of type  $j$  arrives at time period  $t$
- $\phi_{ij}(S)$  : Choice probability of product  $i$  by a customer of type  $j$  from assortment  $S$
- $x_i$  : Remaining inventory of product  $i$

If we offer assortment  $S$  to a customer of type  $j$ , then she purchases product  $i$  with probability

$$\phi_{ij}(S) = \frac{v_{ij}}{1 + \sum_{\ell \in S} v_{\ell j}}$$

- $M$  : Set of customer types
- $N$  : Set of products
- $K$  : Storage size
- $r_i$  : Revenue of product  $i$
- $T$  : Number of time periods in selling horizon
- $\lambda_{jt}$  : Probability that a customer of type  $j$  arrives at time period  $t$
- $\phi_{ij}(S)$  : Choice probability of product  $i$  by a customer of type  $j$  from assortment  $S$
- $x_i$  : Remaining inventory of product  $i$

$$V_t(x) = \sum_{j \in M} \lambda_{jt} \max_{S \subseteq N(x)} \left\{ \sum_{i \in N} \phi_{ij}(S) \left\{ r_i + V_{t+1}(x - e_i) \right\} + \left\{ 1 - \sum_{i \in N} \phi_{ij}(S) \right\} V_{t+1}(x) \right\}$$

assortment customization

$$N(x) = \{ i \in N : x_i > 0 \}$$

- $M$  : Set of customer types
- $N$  : Set of products
- $K$  : Storage size
- $r_i$  : Revenue of product  $i$
- $T$  : Number of time periods in selling horizon
- $\lambda_{jt}$  : Probability that a customer of type  $j$  arrives at time period  $t$
- $\phi_{ij}(S)$  : Choice probability of product  $i$  by a customer of type  $j$  from assortment  $S$
- $x_i$  : Remaining inventory of product  $i$

$$\max_{c \in \mathbb{Z}_+^n} \left\{ V_1(c) : \sum_{i \in N} c_i \leq K \right\}$$

inventory stocking

- $M$  : Set of customer types
- $N$  : Set of products
- $K$  : Storage size
- $r_i$  : Revenue of product  $i$
- $T$  : Number of time periods in selling horizon
- $\lambda_{jt}$  : Probability that a customer of type  $j$  arrives at time period  $t$
- $\phi_{ij}(S)$  : Choice probability of product  $i$  by a customer of type  $j$  from assortment  $S$
- $x_i$  : Remaining inventory of product  $i$

$$\text{OPT} = \max_{c \in \mathbb{Z}_+^n} \left\{ V_1(c) : \sum_{i \in N} c_i \leq K \right\}$$

inventory stocking



## Step 1: Surrogate function construction

Construct a surrogate  $G : \mathbb{Z}_+^n \rightarrow \mathbb{R}_+$  to the initial value function such that

$$G(c) \geq V_1(c) \quad \forall c \in \mathbb{Z}_+^n$$

## Step 2: Inventory stocking

Stock inventory using an  $\alpha$ -approximate solution  $\hat{c}$  to the problem

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G(c) : \sum_{i \in N} c_i \leq K \right\}$$

## Step 3: Assortment customization

Construct a customization policy  $\hat{\pi}$  with total expected revenue

$$\text{Rev}(\hat{c}; \hat{\pi}) \geq \beta G(\hat{c})$$

## Step 1: Surrogate function construction

Construct a surrogate  $G : \mathbb{Z}_+^n \rightarrow \mathbb{R}_+$  to the initial value function such that

$$G(c) \geq V_1(c) \quad \forall c \in \mathbb{Z}_+^n$$

## Step 2: Inventory stocking

Stock inventory using an  $\alpha$ -approximate solution  $\hat{c}$  to the problem

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G(c) : \sum_{i \in N} c_i \leq K \right\}$$

## Step 3: Assortment customization

Construct a customization policy  $\hat{\pi}$  with total expected revenue

$$\text{Rev}(\hat{c}; \hat{\pi}) \geq \beta G(\hat{c})$$

Using the stocking quantities  $\hat{c}$  and subsequently following the assortment personalization policy  $\hat{\pi}$  provides a total expected revenue of at least  $\alpha \beta \text{OPT}$

## Step 1: Surrogate function construction

Construct a surrogate  $G : \mathbb{Z}_+^n \rightarrow \mathbb{R}_+$  to the initial value function such that

$$G(c) \geq V_1(c) \quad \forall c \in \mathbb{Z}_+^n$$

## Step 2: Inventory stocking

Stock inventory using an  $\alpha$ -approximate solution  $\hat{c}$  to the problem

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G(c) : \sum_{i \in N} c_i \leq K \right\}$$

## Step 3: Assortment customization

Construct a customization policy  $\hat{\pi}$  with total expected revenue

$$\text{Rev}(\hat{c}; \hat{\pi}) \geq \beta G(\hat{c})$$

$$\text{Rev}(\hat{c}; \hat{\pi}) \geq \beta G(\hat{c}) \geq \alpha \beta G(c^*) \geq \alpha \beta V_1(c^*) = \alpha \beta \text{OPT}$$

$$c^* = \arg \max_{c \in \mathbb{Z}_+^n} \left\{ V_1(c) : \sum_{i \in N} c_i \leq K \right\}$$

## Step 1: Surrogate function construction

Construct a surrogate  $G : \mathbb{Z}_+^n \rightarrow \mathbb{R}_+$  to the initial value function such that

$$G(c) \geq V_1(c) \quad \forall c \in \mathbb{Z}_+^n$$

## Step 2: Inventory stocking

Stock inventory using an  $\alpha$ -approximate solution  $\hat{c}$  to the problem

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G(c) : \sum_{i \in N} c_i \leq K \right\} \quad \alpha = \frac{1}{2} \left( 1 - \frac{1}{e} \right)$$

## Step 3: Assortment customization

Construct a customization policy  $\hat{\pi}$  with total expected revenue

$$\text{Rev}(\hat{c}; \hat{\pi}) \geq \beta G(\hat{c}) \quad \beta = \frac{1}{2}$$

Can obtain  $\frac{1}{4}(1 - 1/e)$ -approximate solution in poly time

### Step 1: Surrogate function construction

Construct a surrogate  $G : \mathbb{Z}_+^n \rightarrow \mathbb{R}_+$  to the initial value function such that

$$G(c) \geq V_1(c) \quad \forall c \in \mathbb{Z}_+^n$$

### Step 2: Inventory stocking

Stock inventory using an  $\alpha$ -approximate solution  $\hat{c}$  to the problem

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G(c) : \sum_{i \in N} c_i \leq K \right\} \quad \alpha = \frac{1}{2} \left( 1 - \frac{1}{e} \right)$$

### Step 3: Assortment customization

Construct a customization policy  $\hat{\pi}$  with total expected revenue

$$\text{Rev}(\hat{c}; \hat{\pi}) \geq \beta G(\hat{c}) \quad \beta = \frac{1}{2}$$

$z_j(S)$  : Expected number of customers of type  $j$  that are offered assortment  $S$

## LP-based surrogate

$$\begin{aligned} \underline{G}_{LP}(c) = \max & \sum_{j \in M} \sum_{S \subseteq N} \sum_{i \in N} \phi_{ij}(S) r_i z_j(S) \\ \text{st} & \sum_{j \in M} \sum_{S \subseteq N} \phi_{ij}(S) z_j(S) \leq c_i \quad \forall i \in N \\ & \sum_{S \subseteq N} z_j(S) = \sum_{t \in T} \lambda_{jt} \quad \forall j \in M \end{aligned}$$

$z_j(S)$  : Expected number of customers of type  $j$  that are offered assortment  $S$

LP-based surrogate

$$\begin{aligned} \underline{G}_{LP}(c) = \max & \sum_{j \in M} \sum_{S \subseteq N} \sum_{i \in N} \phi_{ij}(S) r_i z_j(S) \\ \text{st} & \sum_{j \in M} \sum_{S \subseteq N} \phi_{ij}(S) z_j(S) \leq c_i \quad \forall i \in N \\ & \sum_{S \subseteq N} z_j(S) = \sum_{t \in T} \lambda_{jt} \quad \forall j \in M \end{aligned}$$

$$G_{LP}(c) \geq V_1(c) \quad \forall c \in \mathbb{Z}_+^n$$

$z_j(S)$  : Expected number of customers of type  $j$  that are offered assortment  $S$

## LP-based surrogate

$$\begin{aligned} \underline{G}_{LP}(c) &= \max \sum_{j \in M} \sum_{S \subseteq N} \sum_{i \in N} \phi_{ij}(S) r_i z_j(S) \\ \text{st} \quad &\sum_{j \in M} \sum_{S \subseteq N} \phi_{ij}(S) z_j(S) \leq c_i \quad \forall i \in N \\ &\sum_{S \subseteq N} z_j(S) = \underbrace{\sum_{t \in T} \lambda_{jt}}_{D_j} \quad \forall j \in M \end{aligned}$$

$$G_{LP}(c) \geq V_1(c) \quad \forall c \in \mathbb{Z}_+^n$$



### Step 1: Surrogate function construction

Construct a surrogate  $G : \mathbb{Z}_+^n \rightarrow \mathbb{R}_+$  to the initial value function such that

$$G(c) \geq V_1(c) \quad \forall c \in \mathbb{Z}_+^n$$

### Step 2: Inventory stocking

Stock inventory using an  $\alpha$ -approximate solution  $\hat{c}$  to the problem

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G(c) : \sum_{i \in N} c_i \leq K \right\} \quad \alpha = \frac{1}{2} \left( 1 - \frac{1}{e} \right)$$

### Step 3: Assortment customization

Construct a customization policy  $\hat{\pi}$  with total expected revenue

$$\text{Rev}(\hat{c}; \hat{\pi}) \geq \beta G(\hat{c}) \quad \beta = \frac{1}{2}$$

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G_{\text{LP}}(c) : \sum_{i \in N} c_i \leq K \right\}$$

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G_{\text{LP}}(c) : \sum_{i \in N} c_i \leq K \right\}$$

NP-hard to approximate within a factor of  $1-1/e$

Consider the function  $G : \mathbb{Z}_+^n \rightarrow \mathbb{R}_+$

Monotone  $c \geq b \implies G(c) \geq G(b)$

Submodular  $c \geq b \implies G(c + e_i) - G(c) \leq G(b + e_i) - G(b)$

$$\max_{\substack{c \in \mathbb{Z}_+^n, \\ \sum_{i \in N} c_i \leq K}} G(c)$$

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G_{\text{LP}}(c) : \sum_{i \in N} c_i \leq K \right\}$$

NP-hard to approximate within a factor of  $1-1/e$

Consider the function  $G : \mathbb{Z}_+^n \rightarrow \mathbb{R}_+$

Monotone  $c \geq b \implies G(c) \geq G(b)$

Submodular  $c \geq b \implies G(c + e_i) - G(c) \leq G(b + e_i) - G(b)$

$G_{\text{LP}}(c)$  is monotone but not submodular in  $c$

Build a monotone and submodular approximation to  $G_{\text{LP}}(c)$

$$\max_{\substack{c \in \mathbb{Z}_+^n, \\ \sum_{i \in N} c_i \leq K}} G(c)$$

## Alternative Reformulation of the Surrogate

$G_{LP}(c) =$

$$\begin{aligned} \max \quad & \sum_{j \in M} \sum_{S \subseteq N} \sum_{i \in N} \phi_{ij}(S) r_i z_j(S) \\ \text{st} \quad & \sum_{j \in M} \sum_{S \subseteq N} \phi_{ij}(S) z_j(S) \leq c_i \quad \forall i \in N \\ & \sum_{S \subseteq N} z_j(S) = D_j \quad \forall j \in M \end{aligned}$$

LP-based surrogate

# Alternative Reformulation of the Surrogate

$G_{LP}(c) =$

$$\begin{aligned} \max \quad & \sum_{j \in M} \sum_{S \subseteq N} \sum_{i \in N} \phi_{ij}(S) r_i z_j(S) \\ \text{st} \quad & \sum_{j \in M} \sum_{S \subseteq N} \phi_{ij}(S) z_j(S) \leq c_i \quad \forall i \in N \\ & \sum_{S \subseteq N} z_j(S) = D_j \quad \forall j \in M \end{aligned}$$

LP-based surrogate

$G_{LP}(c) =$

$$\begin{aligned} \max \quad & \sum_{j \in M} \sum_{i \in N} r_i x_{ij} \\ \text{st} \quad & \sum_{j \in M} x_{ij} \leq c_i \quad \forall i \in N \\ & \sum_{i \in N} x_{ij} + x_{0j} \leq D_j \quad \forall j \in M \\ & x_{ij} \leq v_{ij} x_{0j} \quad \forall i \in N, j \in M \end{aligned}$$

Alternative reformulation

## Submodular Approximation to the Surrogate

$$\begin{aligned} \underline{G_{LP}(c)} &= \max \sum_{j \in M} \sum_{i \in N} r_i x_{ij} \\ \text{st } &\sum_{j \in M} x_{ij} \leq c_i \quad \forall i \in N \\ &\sum_{i \in N} x_{ij} + x_{0j} \leq D_j \quad \forall j \in M \\ &x_{ij} \leq v_{ij} x_{0j} \quad \forall i \in N, j \in M \end{aligned}$$

## Submodular Approximation to the Surrogate

$$\begin{aligned} \underline{G}_{LP}(c) &= \max \sum_{j \in M} \sum_{i \in N} r_i x_{ij} \\ \text{st } \sum_{j \in M} x_{ij} &\leq c_i \quad \forall i \in N \\ \sum_{i \in N} x_{ij} + \underline{x_{0j}} &\leq D_j \quad \forall j \in M \\ x_{ij} &\leq v_{ij} \underline{x_{0j}} \quad \forall i \in N, j \in M \end{aligned}$$

$$\begin{aligned} \underline{G}_{App}(c) &= \max \sum_{j \in M} \sum_{i \in N} r_i x_{ij} \\ \text{st } \sum_{j \in M} x_{ij} &\leq c_i \quad \forall i \in N \\ \sum_{i \in N} x_{ij} &\leq \frac{D_j}{2} \quad \forall j \in M \\ x_{ij} &\leq v_{ij} \frac{D_j}{2} \quad \forall i \in N, j \in M \end{aligned}$$



## Submodular Approximation to the Surrogate

$$\begin{aligned} \underline{G_{LP}}(c) &= \max \sum_{j \in M} \sum_{i \in N} r_i x_{ij} \\ \text{st } \sum_{j \in M} x_{ij} &\leq c_i \quad \forall i \in N \\ \sum_{i \in N} x_{ij} + x_{0j} &\leq D_j \quad \forall j \in M \\ x_{ij} &\leq v_{ij} x_{0j} \quad \forall i \in N, j \in M \end{aligned}$$

$$\begin{aligned} \underline{G_{App}}(c) &= \max \sum_{j \in M} \sum_{i \in N} r_i x_{ij} \\ \text{st } \sum_{j \in M} x_{ij} &\leq c_i \quad \forall i \in N \\ \sum_{i \in N} x_{ij} &\leq \frac{D_j}{2} \quad \forall j \in M \\ x_{ij} &\leq v_{ij} \frac{D_j}{2} \quad \forall i \in N, j \in M \end{aligned}$$

$$\frac{1}{2} G_{LP}(c) \leq G_{App}(c) \leq G_{LP}(c)$$

## Submodular Approximation to the Surrogate

$$\begin{aligned} \underline{G_{LP}}(c) &= \max \sum_{j \in M} \sum_{i \in N} r_i x_{ij} \\ \text{st } \sum_{j \in M} x_{ij} &\leq c_i \quad \forall i \in N \\ \sum_{i \in N} x_{ij} + x_{0j} &\leq D_j \quad \forall j \in M \\ x_{ij} &\leq v_{ij} x_{0j} \quad \forall i \in N, j \in M \end{aligned}$$

$$\begin{aligned} \underline{G_{App}}(c) &= \max \sum_{j \in M} \sum_{i \in N} r_i x_{ij} \\ \text{st } \sum_{j \in M} x_{ij} &\leq c_i \quad \forall i \in N \\ \sum_{i \in N} x_{ij} &\leq \frac{D_j}{2} \quad \forall j \in M \\ x_{ij} &\leq v_{ij} \frac{D_j}{2} \quad \forall i \in N, j \in M \end{aligned}$$

$$\frac{1}{2} G_{LP}(c) \leq G_{App}(c) \leq G_{LP}(c)$$

$G_{App}(c)$  is monotone and submodular in  $c$

# Shelf Allocation

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G_{\text{LP}}(c) : \sum_{i \in N} c_i \leq K \right\}$$

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G_{\text{App}}(c) : \sum_{i \in N} c_i \leq K \right\}$$

### Step 1: Surrogate function construction

Construct a surrogate  $G : \mathbb{Z}_+^n \rightarrow \mathbb{R}_+$  to the initial value function such that

$$G(c) \geq V_1(c) \quad \forall c \in \mathbb{Z}_+^n$$

### Step 2: Inventory stocking

Stock inventory using an  $\alpha$ -approximate solution  $\hat{c}$  to the problem

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G(c) : \sum_{i \in N} c_i \leq K \right\} \quad \alpha = \frac{1}{2} \left( 1 - \frac{1}{e} \right)$$

### Step 3: Assortment customization

Construct a customization policy  $\hat{\pi}$  with total expected revenue

$$\text{Rev}(\hat{c}; \hat{\pi}) \geq \beta G(\hat{c}) \quad \beta = \frac{1}{2}$$

$$\begin{aligned} G_{\text{LP}}(\hat{c}) = \max & \sum_{j \in M} \sum_{S \subseteq N} \sum_{i \in N} \phi_{ij}(S) r_i z_j(S) \\ \text{st} & \sum_{j \in M} \sum_{S \subseteq N} \phi_{ij}(S) z_j(S) \leq \hat{c}_i \quad \forall i \in N \\ & \sum_{S \subseteq N} z_j(S) = D_j \quad \forall j \in M \end{aligned}$$

$$\begin{aligned} G_{\text{LP}}(\hat{c}) &= \max \sum_{j \in M} \sum_{S \subseteq N} \sum_{i \in N} \phi_{ij}(S) r_i z_j(S) \\ \text{st } &\sum_{j \in M} \sum_{S \subseteq N} \phi_{ij}(S) z_j(S) \leq \hat{c}_i \quad \forall i \in N \\ &\sum_{S \subseteq N} \frac{z_j(S)}{D_j} = 1 \quad \forall j \in M \end{aligned}$$

$$\begin{aligned} G_{LP}(\hat{c}) = \max & \sum_{j \in M} \sum_{S \subseteq N} \sum_{i \in N} \phi_{ij}(S) r_i z_j(S) \\ \text{st} & \sum_{j \in M} \sum_{S \subseteq N} \phi_{ij}(S) z_j(S) \leq \hat{c}_i \quad \forall i \in N \\ & \sum_{S \subseteq N} \frac{z_j(S)}{D_j} = 1 \quad \forall j \in M \end{aligned}$$

## Randomized policy

If the remaining inventories at time period  $t$  are  $x$  and a customer of type  $j$  arrives, sample assortment  $S$  with probability  $z_j^*(S)/D_j$

Offer the assortment  $\{i \in S : x_i > 0\}$

$$\begin{aligned} G_{LP}(\hat{c}) = \max & \sum_{j \in M} \sum_{S \subseteq N} \sum_{i \in N} \phi_{ij}(S) r_i z_j(S) \\ \text{st} & \sum_{j \in M} \sum_{S \subseteq N} \phi_{ij}(S) z_j(S) \leq \hat{c}_i \quad \forall i \in N \\ & \sum_{S \subseteq N} \frac{z_j(S)}{D_j} = 1 \quad \forall j \in M \end{aligned}$$

## Randomized policy

If the remaining inventories at time period  $t$  are  $x$  and a customer of type  $j$  arrives, sample assortment  $S$  with probability  $z_j^*(S)/D_j$

Offer the assortment  $\{i \in S : x_i > 0\}$

Total expected revenue of the randomized assortment customization policy satisfies

$$\text{Rev}^{\text{rnd}}(\hat{c}) \geq \frac{1}{2} G_{LP}(\hat{c})$$



## Inventory agnostic policy

If the remaining inventories at time period  $t$  are  $x$  and a customer of type  $j$  arrives, sample assortment  $S$  with probability  $z_j^*(S)/D_j$

Offer the assortment  $S$

If the customer chooses a product with no inventory, she leaves without a purchase

## Inventory agnostic policy

If the remaining inventories at time period  $t$  are  $x$  and a customer of type  $j$  arrives, sample assortment  $S$  with probability  $z_j^*(S)/D_j$

Offer the assortment  $S$

If the customer chooses a product with no inventory, she leaves without a purchase

$$\text{Rev}^{\text{rnd}}(\hat{c}) \geq \text{Rev}^{\text{agn}}(\hat{c})$$

## Inventory agnostic policy

If the remaining inventories at time period  $t$  are  $x$  and a customer of type  $j$  arrives, sample assortment  $S$  with probability  $z_j^*(S)/D_j$

Offer the assortment  $S$

If the customer chooses a product with no inventory, she leaves without a purchase

Demand for product  $i$  at time period  $t$  under the inventory agnostic policy

$$X_{it}^{\text{agn}} \sim \text{Bernoulli} \left( \sum_{j \in M} \sum_{S \subseteq N} \lambda_{jt} \frac{z_j^*(S)}{D_j} \phi_{ij}(S) \right)$$

$$\text{Rev}^{\text{rnd}}(\hat{c}) \geq \text{Rev}^{\text{agn}}(\hat{c})$$

## Inventory agnostic policy

If the remaining inventories at time period  $t$  are  $x$  and a customer of type  $j$  arrives, sample assortment  $S$  with probability  $z_j^*(S)/D_j$

Offer the assortment  $S$

If the customer chooses a product with no inventory, she leaves without a purchase

Demand for product  $i$  at time period  $t$  under the inventory agnostic policy

$$X_{it}^{\text{agn}} \sim \text{Bernoulli} \left( \sum_{j \in M} \sum_{S \subseteq N} \lambda_{jt} \frac{z_j^*(S)}{D_j} \phi_{ij}(S) \right)$$

$$\text{Rev}^{\text{rnd}}(\hat{c}) \geq \text{Rev}^{\text{agn}}(\hat{c}) = \sum_{i \in N} \sum_{t \in T} r_i \mathbb{E}\{X_{it}^{\text{agn}}\} - \sum_{i \in N} r_i \mathbb{E} \left[ \sum_{t \in T} X_{it}^{\text{agn}} - \hat{c}_i \right]^+$$

## Inventory agnostic policy

If the remaining inventories at time period  $t$  are  $x$  and a customer of type  $j$  arrives, sample assortment  $S$  with probability  $z_j^*(S)/D_j$

Offer the assortment  $S$

If the customer chooses a product with no inventory, she leaves without a purchase

Demand for product  $i$  at time period  $t$  under the inventory agnostic policy

$$X_{it}^{\text{agn}} \sim \text{Bernoulli} \left( \sum_{j \in M} \sum_{S \subseteq N} \lambda_{jt} \frac{z_j^*(S)}{D_j} \phi_{ij}(S) \right)$$

$$\text{Rev}^{\text{rnd}}(\hat{c}) \geq \text{Rev}^{\text{agn}}(\hat{c}) = \sum_{i \in N} \sum_{t \in T} r_i \mathbb{E}\{X_{it}^{\text{agn}}\} - \sum_{i \in N} r_i \mathbb{E} \left[ \sum_{t \in T} X_{it}^{\text{agn}} - \hat{c}_i \right]^+$$

$$\begin{array}{ccc} \parallel & & \wedge \\ \text{GLP}(\hat{c}) & & \frac{1}{2} \text{GLP}(\hat{c}) \end{array}$$

## Inventory agnostic policy

If the remaining inventories at time period  $t$  are  $x$  and a customer of type  $j$  arrives, sample assortment  $S$  with probability  $z_j^*(S)/D_j$

Offer the assortment  $S$

If the customer chooses a product with no inventory, she leaves without a purchase

Demand for product  $i$  at time period  $t$  under the inventory agnostic policy

$$X_{it}^{\text{agn}} \sim \text{Bernoulli} \left( \sum_{j \in M} \sum_{S \subseteq N} \lambda_{jt} \frac{z_j^*(S)}{D_j} \phi_{ij}(S) \right)$$

$$\text{Rev}^{\text{rnd}}(\hat{c}) \geq \text{Rev}^{\text{agn}}(\hat{c}) = \sum_{i \in N} \sum_{t \in T} r_i \mathbb{E}\{X_{it}^{\text{agn}}\} - \sum_{i \in N} r_i \mathbb{E} \left[ \sum_{t \in T} X_{it}^{\text{agn}} - \hat{c}_i \right]^+ \geq \frac{1}{2} G_{\text{LP}}(\hat{c})$$

$$\begin{array}{ccc} & \parallel & \wedge \\ & G_{\text{LP}}(\hat{c}) & \frac{1}{2} G_{\text{LP}}(\hat{c}) \end{array}$$

Under general choice models,  $(1 - 3\sqrt[3]{n/K})$ -approximation in poly time

Budget constraints of the form  $\sum_{i \in N} w_i c_i \leq K$

Constant-factor approximation under general choice models

Coordinated inventory stocking and assortment customization  
Bai, El Housni, Rusmevichientong, Topaloglu