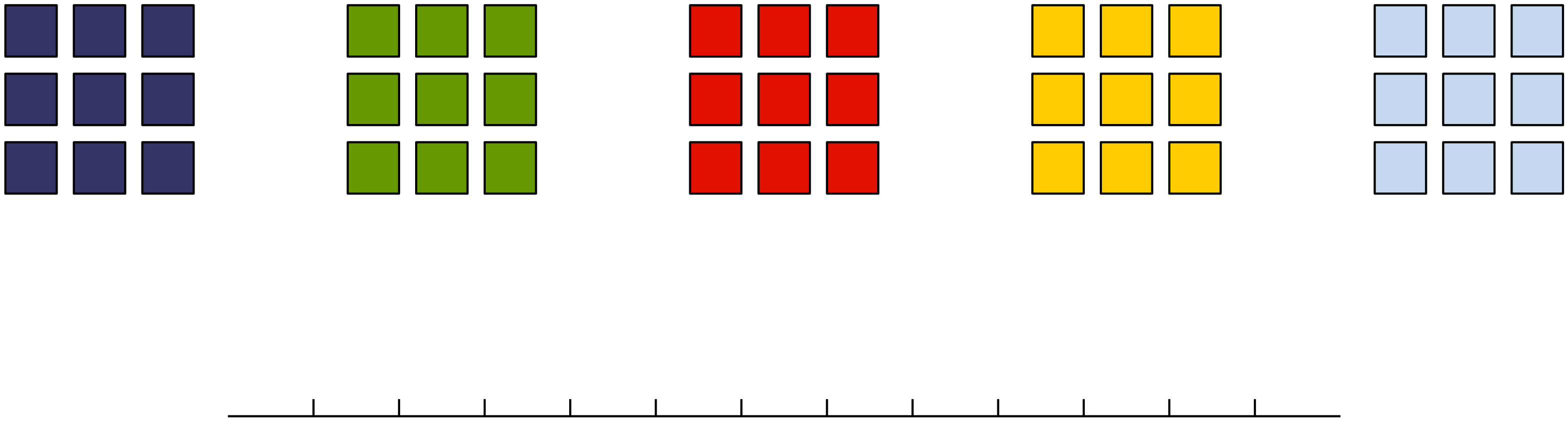
Coordinated Inventory Stocking and Assortment Customization

Huseyin Topaloglu
Cornell Tech

Yicheng Bai, Omar El Housni, Paat Rusmevichientong

Joint Inventory Stocking and Assortment Customization



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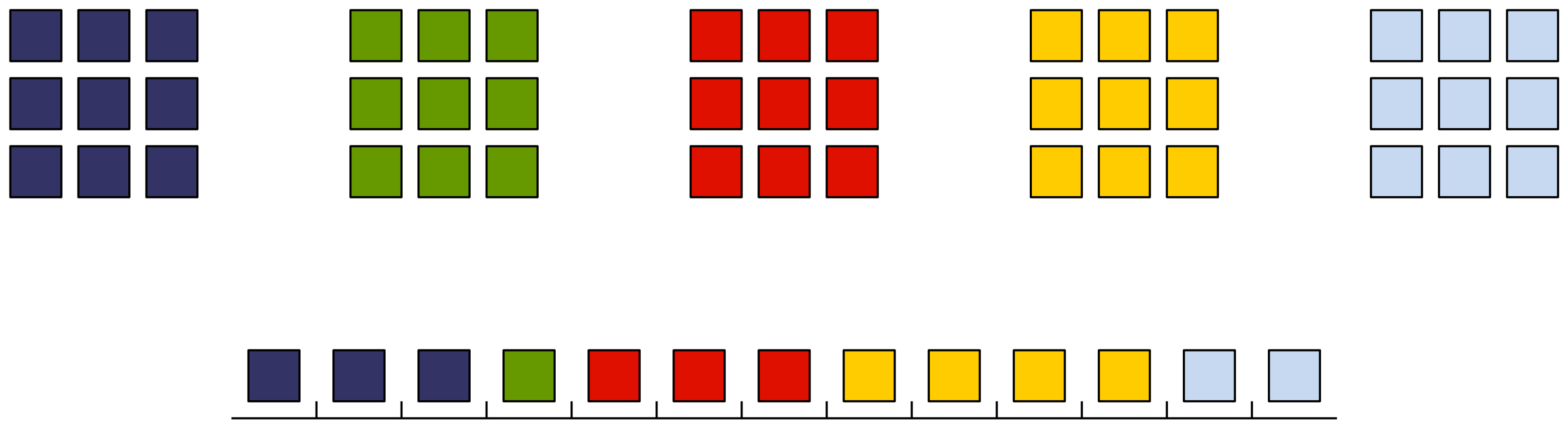

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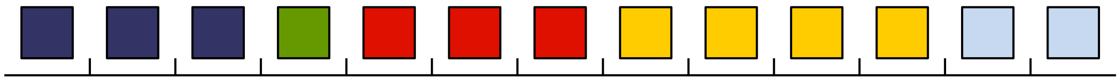

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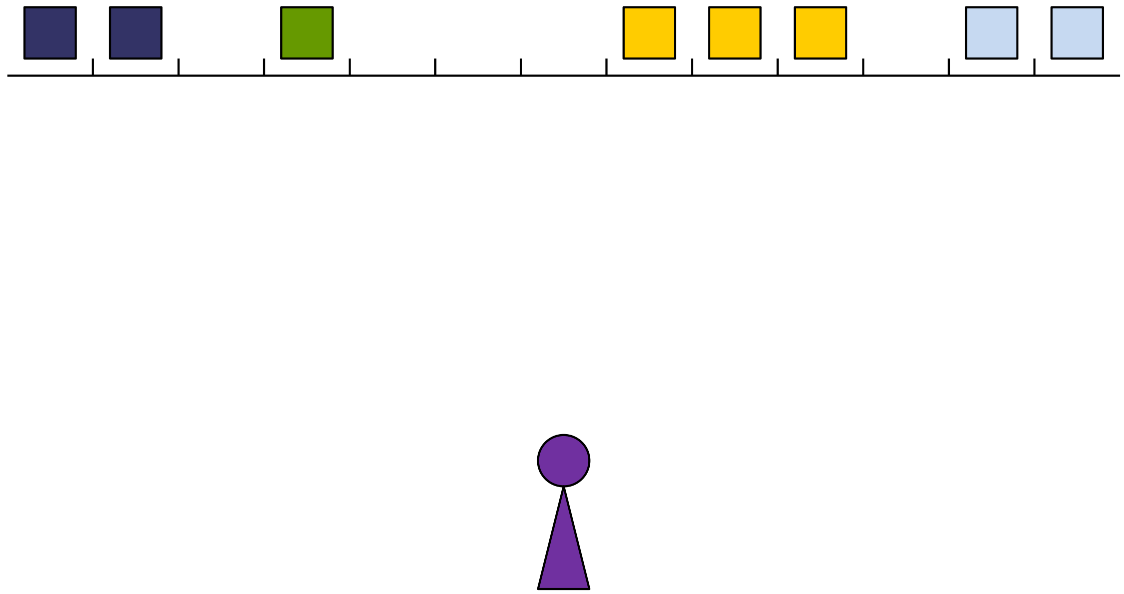

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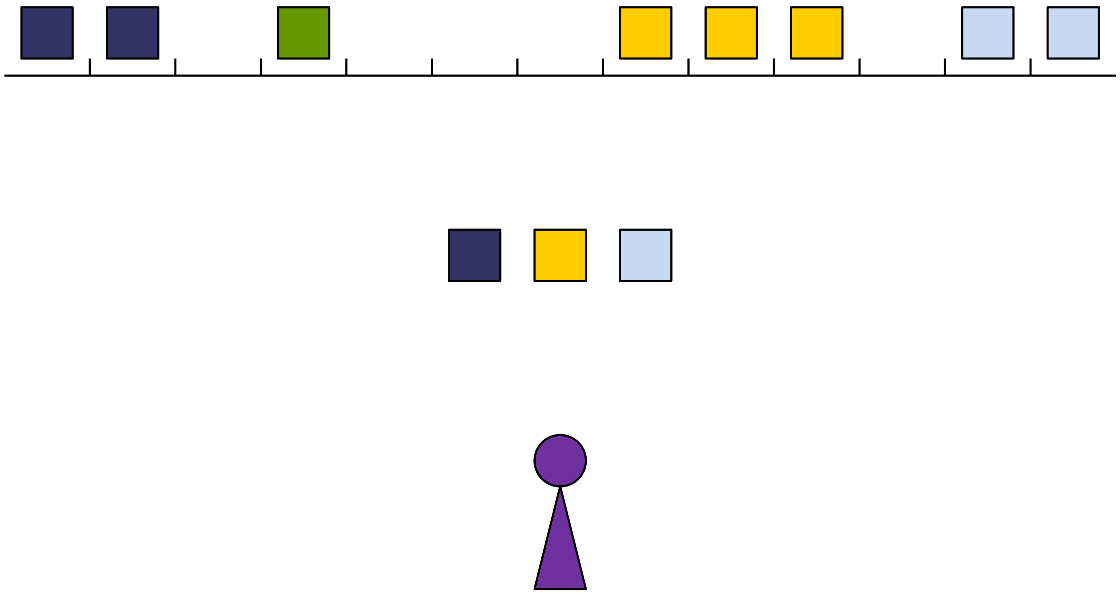

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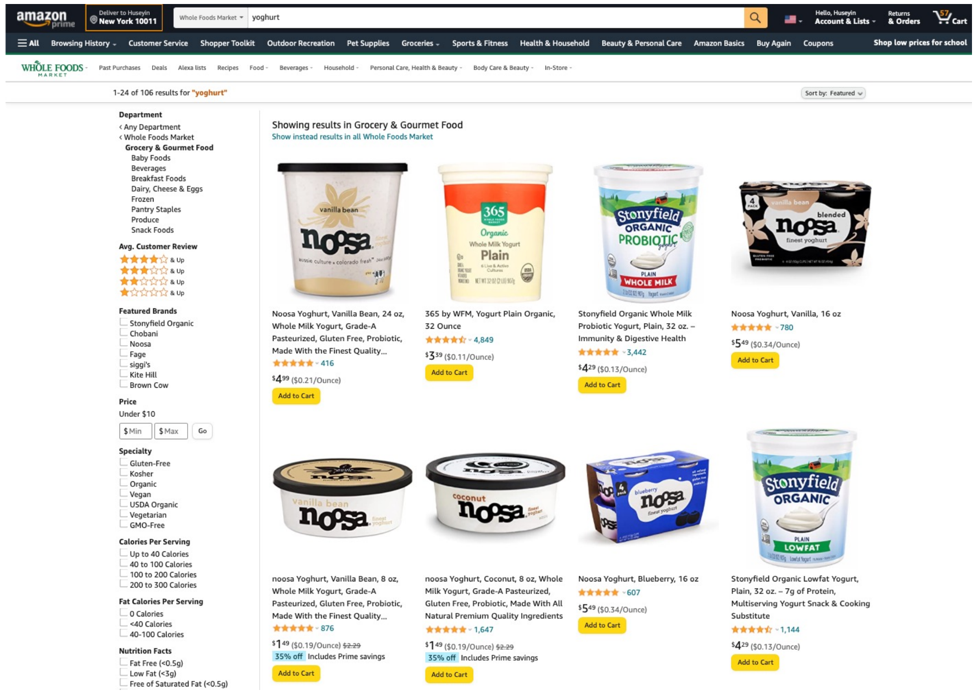

Joint Inventory Stocking and Assortment Customization

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115
FAGE
Total
OATLY!
11
unsweetened vanilla
SO DELICIOUS
coconutmilk
Stonyfield
ORGANIC
LOWFAT
Plain
49
noosa
152
Chobani Complete
36
Chobani

Joint Inventory Stocking and Assortment Customization

Optimization Model

M	:	Set of customer types
N	:	Set of products
K	:	Storage size
r_i	:	Revenue of product i
T	:	Number of time periods in selling horizon
λ_{jt}	:	Probability that a customer of type j arrives at time period t
$\phi_{ij}(S)$	:	Choice probability of product i by a customer of type j from assortment S
x_i	:	Remaining inventory of product i

Optimization Model

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λ_{jt}	:	Probability that a customer of type j arrives at time period t
$\phi_{ij}(S)$	:	Choice probability of product i by a customer of type j from assortment S
x_i	:	Remaining inventory of product i

If we offer assortment S to a customer of type j, then she purchases product i with probability

$$\phi_{ij}(S) = \frac{v_{ij}}{1 + \sum_{\ell \in S} v_{\ell j}}$$

Optimization Model

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x_i	:	Remaining inventory of product i

$$V_t(x) = \sum_{j \in M} \lambda_{jt} \max_{S \subseteq N(x)} \left\{ \sum_{i \in N} \phi_{ij}(S) \left\{ r_i + V_{t+1}(x - e_i) \right\} + \left\{ 1 - \sum_{i \in N} \phi_{ij}(S) \right\} V_{t+1}(x) \right\}$$

assortment customization

$$N(x) = \left\{ i \in N \ : \ x_i > 0 \right\}$$

Optimization Model

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x_i	:	Remaining inventory of product i

$$\max_{c \in \mathbb{Z}_+^n} \left\{ V_1(c) \ : \ \sum_{i \in N} c_i \leq K \right\}$$

inventory stocking

Optimization Model

M	:	Set of customer types
N	:	Set of products
K	:	Storage size
r_i	:	Revenue of product i
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λ_{jt}	:	Probability that a customer of type j arrives at time period t
$\phi_{ij}(S)$	:	Choice probability of product i by a customer of type j from assortment S
x_i	:	Remaining inventory of product i

$$\text{OPT} = \max_{c \in \mathbb{Z}_+^n} \left\{ V_1(c) \ : \ \sum_{i \in N} c_i \leq K \right\}$$

inventory stocking

Step 1: Surrogate function construction

Construct a surrogate $G : \mathbb{Z}_+^n \to \mathbb{R}_+$ to the initial value function such that

$$G(c) \geq V_1(c) \qquad \forall\, c \in \mathbb{Z}_+^n$$

Step 2: Inventory stocking

Stock inventory using an α-approximate solution $\widehat{c}$ to the problem

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G(c) \;:\; \sum_{i \in N} c_i \leq K \right\}$$

Step 3: Assortment customization

Construct a customization policy $\widehat{\pi}$ with total expected revenue

$$\mathrm{Rev}(\widehat{c}; \widehat{\pi}) \;\geq\; \beta\, G(\widehat{c})$$

Approximation Framework

Step 1: Surrogate function construction

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$$\mathrm{Rev}(\widehat{c}; \widehat{\pi}) \;\geq\; \beta\, G(\widehat{c})$$

Using the stocking quantities $\widehat{c}$ and subsequently following the assortment personalization policy $\widehat{\pi}$ provides a total expected revenue of at least $\alpha\,\beta\,\mathrm{OPT}$

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$$\mathrm{Rev}(\widehat{c}; \widehat{\pi}) \;\geq\; \beta\, G(\widehat{c}) \;\geq\; \alpha\, \beta\, G(c^*) \;\geq\; \alpha\, \beta\, V_1(c^*) \;=\; \alpha\, \beta\, \mathrm{OPT}$$

$$c^* = \arg\max_{c \in \mathbb{Z}_+^n} \left\{ V_1(c) \;:\; \sum_{i \in N} c_i \leq K \right\}$$

Approximation Framework

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$$\max_{c \in \mathbb{Z}_+^n} \left\{ G(c) \;:\; \sum_{i \in N} c_i \leq K \right\} \qquad \alpha = \tfrac{1}{2}(1 - \tfrac{1}{e})$$

Step 3: Assortment customization

Construct a customization policy $\widehat{\pi}$ with total expected revenue

$$\mathrm{Rev}(\widehat{c}; \widehat{\pi}) \;\geq\; \beta\, G(\widehat{c}) \qquad \beta = \tfrac{1}{2}$$

Can obtain $\frac{1}{4}(1 - 1/e)$-approximate solution in poly time

Outline

Step 1: Surrogate function construction

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Surrogate Construction

$z_j(S)$: Expected number of customers of type j that are offered assortment S

LP-based surrogate

$$
\begin{aligned}
\underline{G_{\mathrm{LP}}(c)} \quad = \quad \max \quad & \sum_{j \in M} \sum_{S \subseteq N} \sum_{i \in N} \phi_{ij}(S)\, r_i\, z_j(S) \\
\text{st} \quad & \sum_{j \in M} \sum_{S \subseteq N} \phi_{ij}(S)\, z_j(S) \leq c_i \quad \forall\, i \in N \\
& \sum_{S \subseteq N} z_j(S) = \sum_{t \in T} \lambda_{jt} \quad \forall\, j \in M
\end{aligned}
$$

Gallego et al. (2004), Liu and van Ryzin (2008)

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$$G_{\mathrm{LP}}(c) \ge V_1(c) \qquad \forall\, c \in \mathbb{Z}_+^n$$

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& \sum_{S\subseteq N} z_j(S) = \underbrace{\sum_{t\in T} \lambda_{jt}}_{D_j} \quad \forall\, j \in M
\end{aligned}
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G_{\mathrm{LP}}(c) \ge V_1(c) \qquad \forall\, c \in \mathbb{Z}_+^n
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Stock inventory using an α-approximate solution $\widehat{c}$ to the problem

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G(c) \ : \ \sum_{i \in N} c_i \leq K \right\} \qquad \alpha = \frac{1}{2}\left(1 - \frac{1}{e}\right)$$

Step 3: Assortment customization

Construct a customization policy $\widehat{\pi}$ with total expected revenue

$$\text{Rev}(\widehat{c}; \widehat{\pi}) \ \geq \ \beta\, G(\widehat{c}) \qquad \beta = \frac{1}{2}$$

Submodularity

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G_{\mathrm{LP}}(c) \ : \ \sum_{i \in N} c_i \leq K \right\}$$

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G_{\mathrm{LP}}(c) \;:\; \sum_{i \in N} c_i \le K \right\}$$

NP-hard to approximate within a factor of 1–1/e

Consider the function $G : \mathbb{Z}_+^n \to \mathbb{R}_+$

Monotone $c \ge b \implies G(c) \ge G(b)$

Submodular $c \ge b \implies G(c + e_i) - G(c) \le G(b + e_i) - G(b)$

Nemhauser et al. (1978), Lee et al. (2009), Soma and Yoshida (2018)

$$\max_{\substack{c \in \mathbb{Z}_+^n, \\ \sum_{i \in N} c_i \le K}} G(c)$$

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G_{\mathrm{LP}}(c) \ : \ \sum_{i \in N} c_i \leq K \right\}$$

NP-hard to approximate within a factor of 1–1/e

Consider the function $G : \mathbb{Z}_+^n \to \mathbb{R}_+$

Monotone $c \geq b \implies G(c) \geq G(b)$

Submodular $c \geq b \implies G(c + e_i) - G(c) \leq G(b + e_i) - G(b)$

$G_{\mathrm{LP}}(c)$ is monotone but not submodular in c

Build a monotone and submodular approximation to $G_{\mathrm{LP}}(c)$

$$\max_{\substack{c \in \mathbb{Z}_+^n, \\ \sum_{i \in N} c_i \leq K}} G(c)$$

Nemhauser et al. (1978), Lee et al. (2009), Soma and Yoshida (2018)

Alternative Reformulation of the Surrogate

$G_{\text{LP}}(c) =$

$$\begin{aligned} \max \quad & \sum_{j \in M} \sum_{S \subseteq N} \sum_{i \in N} \phi_{ij}(S)\, r_i\, z_j(S) \\ \text{st} \quad & \sum_{j \in M} \sum_{S \subseteq N} \phi_{ij}(S)\, z_j(S) \leq c_i \quad \forall\, i \in N \\ & \sum_{S \subseteq N} z_j(S) = D_j \quad \forall\, j \in M \end{aligned}$$

LP-based surrogate

Alternative Reformulation of the Surrogate

$G_{\text{LP}}(c) =$

$$\begin{aligned} \max \quad & \sum_{j\in M} \sum_{S\subseteq N} \sum_{i\in N} \phi_{ij}(S)\, r_i\, z_j(S) \\ \text{st} \quad & \sum_{j\in M} \sum_{S\subseteq N} \phi_{ij}(S)\, z_j(S) \le c_i \quad \forall\, i \in N \\ & \sum_{S\subseteq N} z_j(S) = D_j \quad \forall\, j \in M \end{aligned}$$

LP-based surrogate

$G_{\text{LP}}(c) =$

$$\begin{aligned} \max \quad & \sum_{j\in M} \sum_{i\in N} r_i\, x_{ij} \\ \text{st} \quad & \sum_{j\in M} x_{ij} \le c_i \quad \forall\, i \in N \\ & \sum_{i\in N} x_{ij} + x_{0j} \le D_j \quad \forall\, j \in M \\ & x_{ij} \le v_{ij}\, x_{0j} \quad \forall\, i \in N,\ j \in M \end{aligned}$$

Alternative reformulation

Topaloglu (2013), Gallego et al. (2015), Feldman and Topaloglu (2017), Cao et al. (2021), Cao et al. (2022)

$$
\begin{aligned}
G_{\mathrm{LP}}(c) = \max \quad & \sum_{j \in M} \sum_{i \in N} r_i \, x_{ij} \\
\text{st} \quad & \sum_{j \in M} x_{ij} \le c_i \quad \forall\, i \in N \\
& \sum_{i \in N} x_{ij} + x_{0j} \le D_j \quad \forall\, j \in M \\
& x_{ij} \le v_{ij} \, x_{0j} \quad \forall\, i \in N,\ j \in M
\end{aligned}
$$

Submodular Approximation to the Surrogate

$$
\begin{aligned}
G_{\mathrm{LP}}(c) = \max \quad & \sum_{j \in M} \sum_{i \in N} r_i \, x_{ij} \\
\text{st} \quad & \sum_{j \in M} x_{ij} \le c_i \quad \forall \, i \in N \\
& \sum_{i \in N} x_{ij} + x_{0j} \le D_j \quad \forall \, j \in M \\
& x_{ij} \le v_{ij} \, x_{0j} \quad \forall \, i \in N, \; j \in M
\end{aligned}
$$

$$
\begin{aligned}
G_{\mathrm{App}}(c) = \max \quad & \sum_{j \in M} \sum_{i \in N} r_i \, x_{ij} \\
\text{st} \quad & \sum_{j \in M} x_{ij} \le c_i \quad \forall \, i \in N \\
& \sum_{i \in N} x_{ij} \le \frac{D_j}{2} \quad \forall \, j \in M \\
& x_{ij} \le v_{ij} \, \frac{D_j}{2} \quad \forall \, i \in N, \; j \in M
\end{aligned}
$$

Submodular Approximation to the Surrogate

$$G_{\mathrm{LP}}(c) = \max \sum_{j \in M} \sum_{i \in N} r_i \, x_{ij}$$
$$\text{st} \sum_{j \in M} x_{ij} \le c_i \quad \forall\, i \in N$$
$$\sum_{i \in N} x_{ij} + x_{0j} \le D_j \quad \forall\, j \in M$$
$$x_{ij} \le v_{ij} \, x_{0j} \quad \forall\, i \in N,\ j \in M$$

$$G_{\mathrm{App}}(c) = \max \sum_{j \in M} \sum_{i \in N} r_i \, x_{ij}$$
$$\text{st} \sum_{j \in M} x_{ij} \le c_i \quad \forall\, i \in N$$
$$\sum_{i \in N} x_{ij} \le \frac{D_j}{2} \quad \forall\, j \in M$$
$$x_{ij} \le v_{ij} \, \frac{D_j}{2} \quad \forall\, i \in N,\ j \in M$$

$$\tfrac{1}{2}\, G_{\mathrm{LP}}(c) \;\le\; G_{\mathrm{App}}(c) \;\le\; G_{\mathrm{LP}}(c)$$

Submodular Approximation to the Surrogate

$$G_{\mathrm{LP}}(c) = \max \sum_{j \in M} \sum_{i \in N} r_i \, x_{ij}$$
$$\text{st} \sum_{j \in M} x_{ij} \leq c_i \quad \forall\, i \in N$$
$$\sum_{i \in N} x_{ij} + x_{0j} \leq D_j \quad \forall\, j \in M$$
$$x_{ij} \leq v_{ij} \, x_{0j} \quad \forall\, i \in N,\ j \in M$$

$$G_{\mathrm{App}}(c) = \max \sum_{j \in M} \sum_{i \in N} r_i \, x_{ij}$$
$$\text{st} \sum_{j \in M} x_{ij} \leq c_i \quad \forall\, i \in N$$
$$\sum_{i \in N} x_{ij} \leq \tfrac{D_j}{2} \quad \forall\, j \in M$$
$$x_{ij} \leq v_{ij} \, \tfrac{D_j}{2} \quad \forall\, i \in N,\ j \in M$$

$$\tfrac{1}{2}\, G_{\mathrm{LP}}(c) \;\leq\; G_{\mathrm{App}}(c) \;\leq\; G_{\mathrm{LP}}(c)$$

$G_{\mathrm{App}}(c)$ is monotone and submodular in c

Shelf Allocation

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G_{\text{LP}}(c) \ : \ \sum_{i \in N} c_i \leq K \right\}$$

$$\max_{c \in \mathbb{Z}_+^n} \left\{ G_{\text{App}}(c) \ : \ \sum_{i \in N} c_i \leq K \right\}$$

Outline

Step 1: Surrogate function construction

Construct a surrogate $G : \mathbb{Z}_+^n \to \mathbb{R}_+$ to the initial value function such that

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Step 2: Inventory stocking

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Step 3: Assortment customization

Construct a customization policy $\widehat{\pi}$ with total expected revenue

$$\mathrm{Rev}(\widehat{c}; \widehat{\pi}) \;\geq\; \beta\, G(\widehat{c}) \qquad \beta = \frac{1}{2}$$

Randomized Assortment Customization Policy

$$
\begin{aligned}
G_{\mathrm{LP}}(\widehat{c}) \quad = \quad \max \quad & \sum_{j\in M} \sum_{S\subseteq N} \sum_{i\in N} \phi_{ij}(S)\, r_i\, z_j(S) \\
\text{st} \quad & \sum_{j\in M} \sum_{S\subseteq N} \phi_{ij}(S)\, z_j(S) \le \widehat{c}_i \quad \forall\, i \in N \\
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& \sum_{S\subseteq N} \frac{z_j(S)}{D_j} = 1 \quad \forall\, j \in M
\end{aligned}
$$

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\end{aligned}
$$

Randomized policy

If the remaining inventories at time period t are x *and* a customer of type j arrives, sample assortment S with probability $z_j^*(S)/D_j$

Offer the assortment $\{i \in S : x_i > 0\}$

Randomized Assortment Customization Policy

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\begin{aligned}
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Randomized policy

If the remaining inventories at time period t are x *and* a customer of type j arrives, sample assortment S with probability $z_j^*(S)/D_j$

Offer the assortment $\{i \in S : x_i > 0\}$

Total expected revenue of the randomized assortment customization policy satisfies

$$
\mathrm{Rev}^{\mathrm{rnd}}(\widehat{c}) \;\ge\; \frac{1}{2}\, G_{\mathrm{LP}}(\widehat{c})
$$

Golrezaei et al. (2014), Rusmevichientong et al. (2020), Bumpensanti and Wang (2020), Ma et al. (2020), Feng et al. (2020), Baek and Ma (2022)

Inventory agnostic policy

If the remaining inventories at time period t are x *and* a customer of type j arrives, sample assortment S with probability $z_j^*(S)/D_j$

Offer the assortment S

If the customer chooses a product with no inventory, she leaves without a purchase

Randomized Assortment Customization Policy

Inventory agnostic policy

If the remaining inventories at time period t are x *and* a customer of type j arrives, sample assortment S with probability $z_j^*(S)/D_j$

Offer the assortment S

If the customer chooses a product with no inventory, she leaves without a purchase

$$\text{Rev}^{\text{rnd}}(\hat{c}) \;\geq\; \text{Rev}^{\text{agn}}(\hat{c})$$

Randomized Assortment Customization Policy

Inventory agnostic policy

If the remaining inventories at time period t are x *and* a customer of type j arrives, sample assortment S with probability $z_j^*(S)/D_j$

Offer the assortment S

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Demand for product i at time period t under the inventory agnostic policy

$$X_{it}^{\text{agn}} \sim \text{Bernoulli}\left(\sum_{j \in M} \sum_{S \subseteq N} \lambda_{jt} \frac{z_j^*(S)}{D_j} \phi_{ij}(S) \right)$$

$$\text{Rev}^{\text{rnd}}(\widehat{c}) \;\geq\; \text{Rev}^{\text{agn}}(\widehat{c})$$

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Demand for product i at time period t under the inventory agnostic policy

$$X_{it}^{\text{agn}} \sim \text{Bernoulli}\left(\sum_{j \in M} \sum_{S \subseteq N} \lambda_{jt} \, \frac{z_j^*(S)}{D_j} \, \phi_{ij}(S) \right)$$

$$\text{Rev}^{\text{rnd}}(\widehat{c}) \;\geq\; \text{Rev}^{\text{agn}}(\widehat{c}) \;=\; \sum_{i \in N} \sum_{t \in T} r_i \, \mathbb{E}\{X_{it}^{\text{agn}}\} - \sum_{i \in N} r_i \, \mathbb{E}\left[\sum_{t \in T} X_{it}^{\text{agn}} - \widehat{c}_i \right]^+$$

Inventory agnostic policy

If the remaining inventories at time period t are x *and* a customer of type j arrives, sample assortment S with probability $z_j^*(S)/D_j$

Offer the assortment S

If the customer chooses a product with no inventory, she leaves without a purchase

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$$\sum_{i \in N} \sum_{t \in T} r_i \, \mathbb{E}\{X_{it}^{\text{agn}}\} = G_{\text{LP}}(\widehat{c}), \qquad \sum_{i \in N} r_i \, \mathbb{E}\left[\sum_{t \in T} X_{it}^{\text{agn}} - \widehat{c}_i \right]^+ \leq \frac{1}{2} G_{\text{LP}}(\widehat{c})$$

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Conclusions

Under general choice models, $(1 - 3\sqrt[3]{n/K})$-approximation in poly time

Budget constraints of the form $\sum_{i \in N} w_i \, c_i \leq K$

Constant-factor approximation under general choice models

Coordinated inventory stocking and assortment customization
Bai, El Housni, Rusmevichientong, Topaloglu