



NGB-LNMB conference, January 2025

Dike height optimization

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Dick den Hertog (University of Amsterdam)





INFORMS Franz Edelman Award 2013





Background





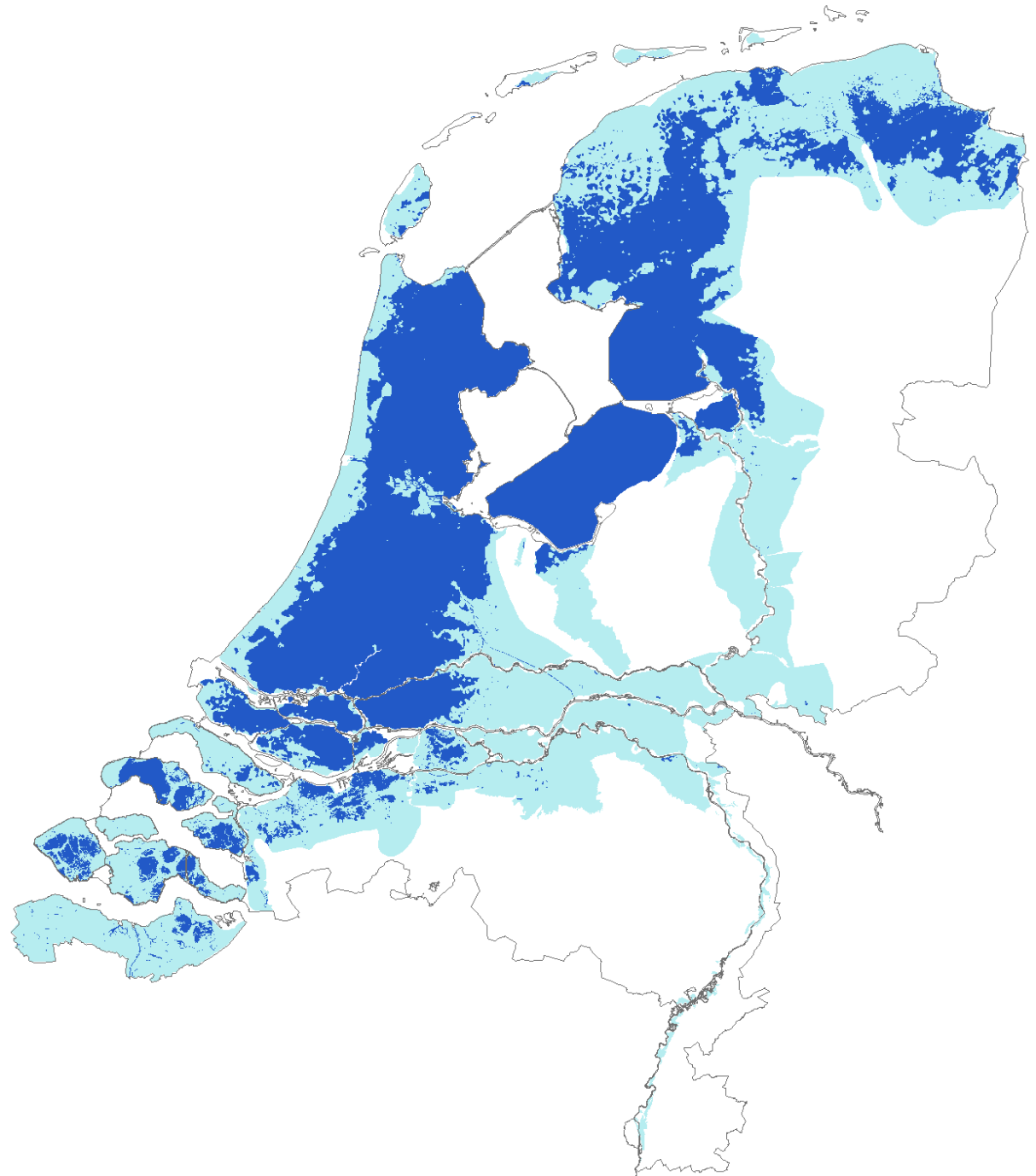
Flood prone areas



Below sea level



Above sea level









Flood in 1953

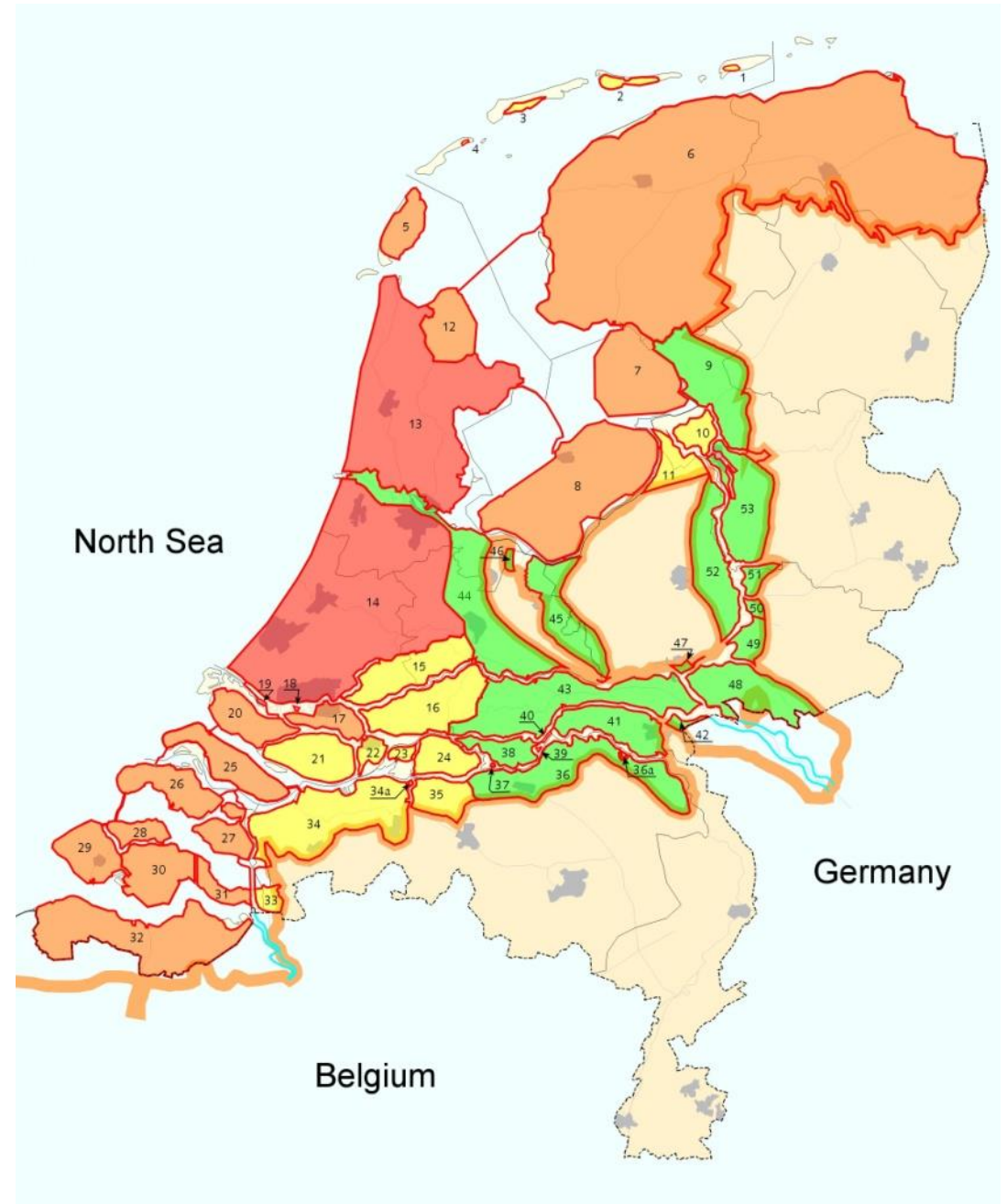


Flood protection standards

	1 / 1250 per year
	1 / 2000 per year
	1 / 4000 per year
	1 / 10,000 per year

Former legal flood
protection standards

53 dike ring areas





Critical situation in 1995





Methodology



THE FRANZ EDELMAN AWARD
Achievement in Operations Research

Economically Efficient Standards to Protect the Netherlands Against Flooding

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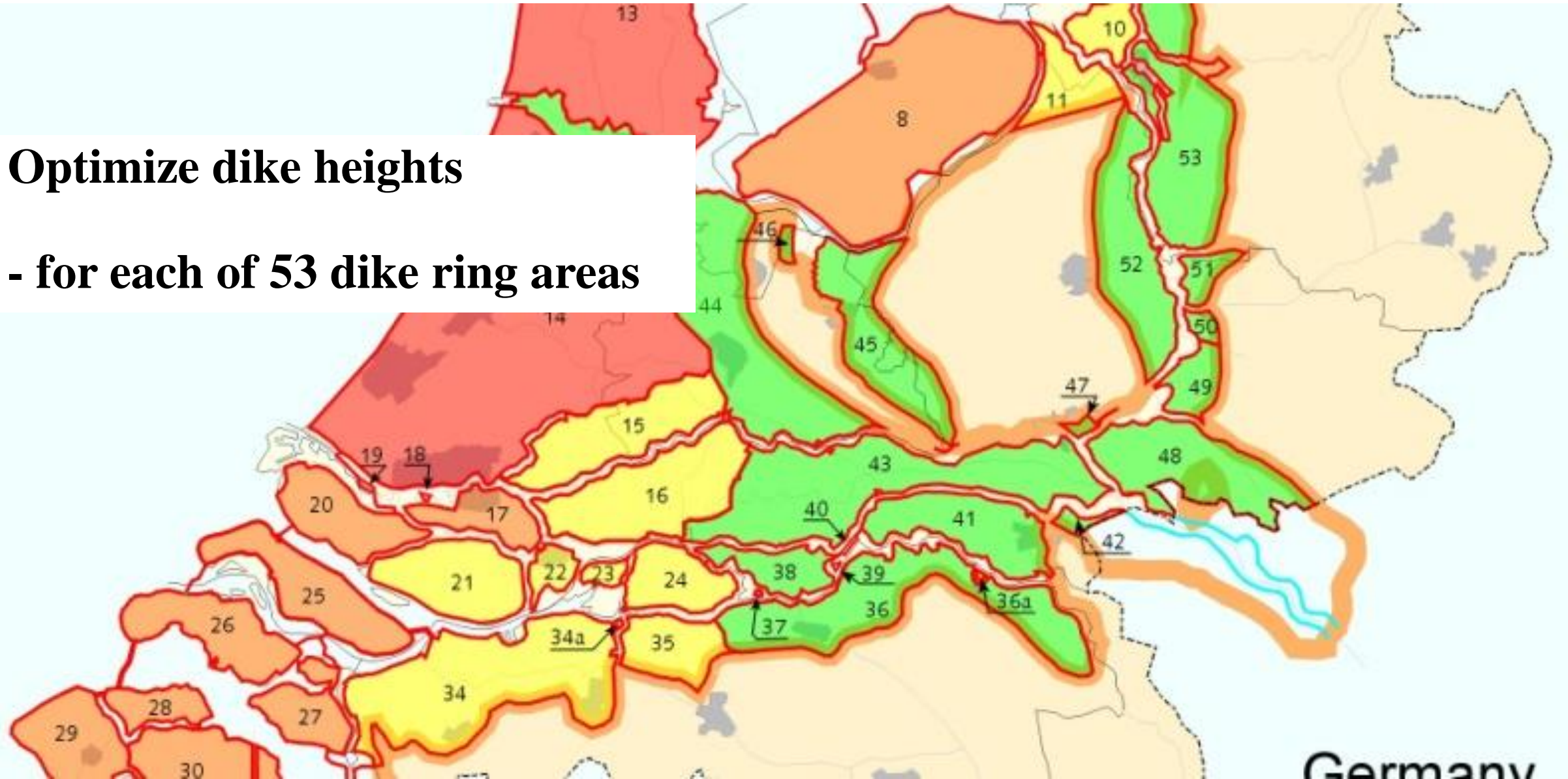
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Wim Kuijken

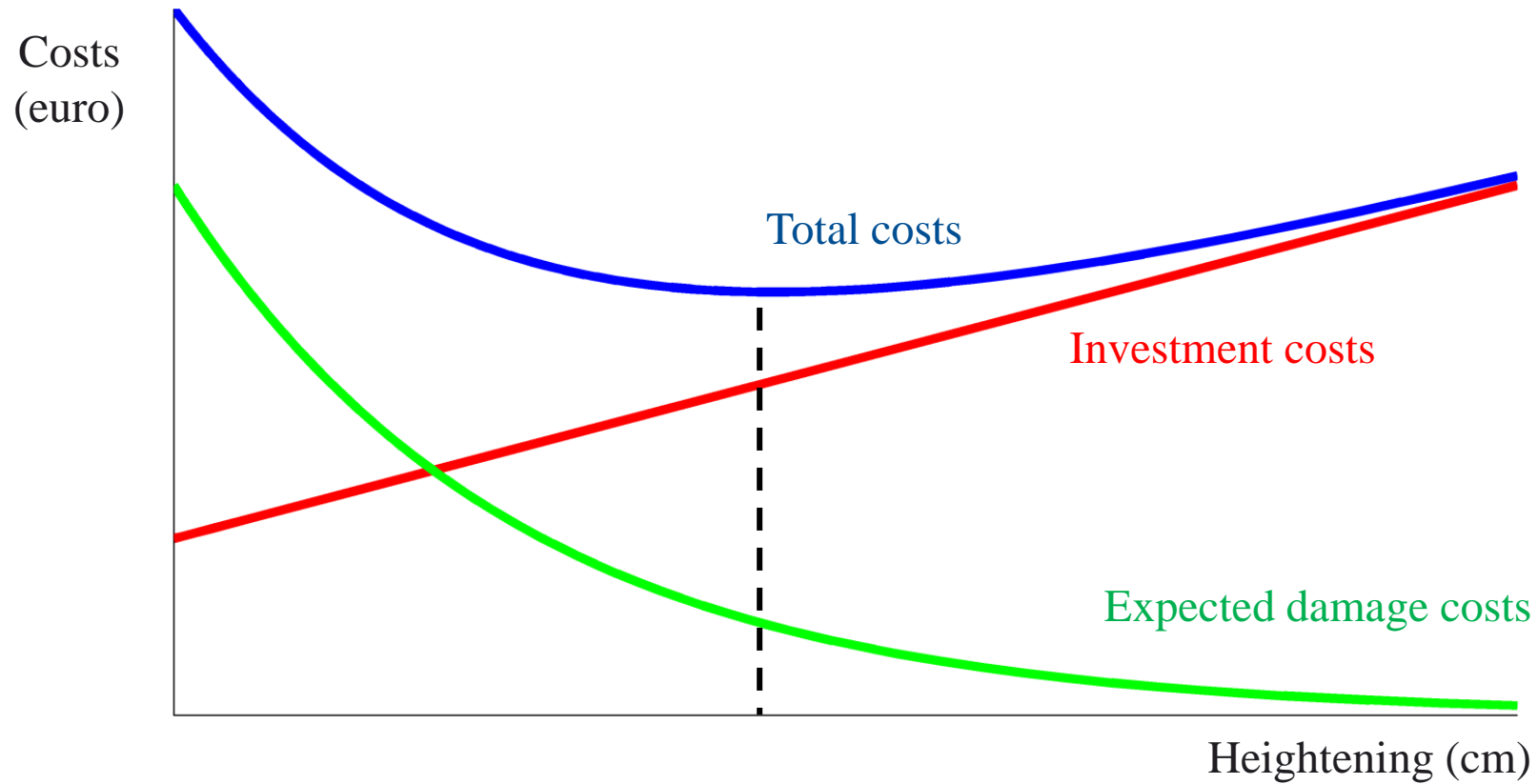
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Optimize dike heights
- for each of 53 dike ring areas



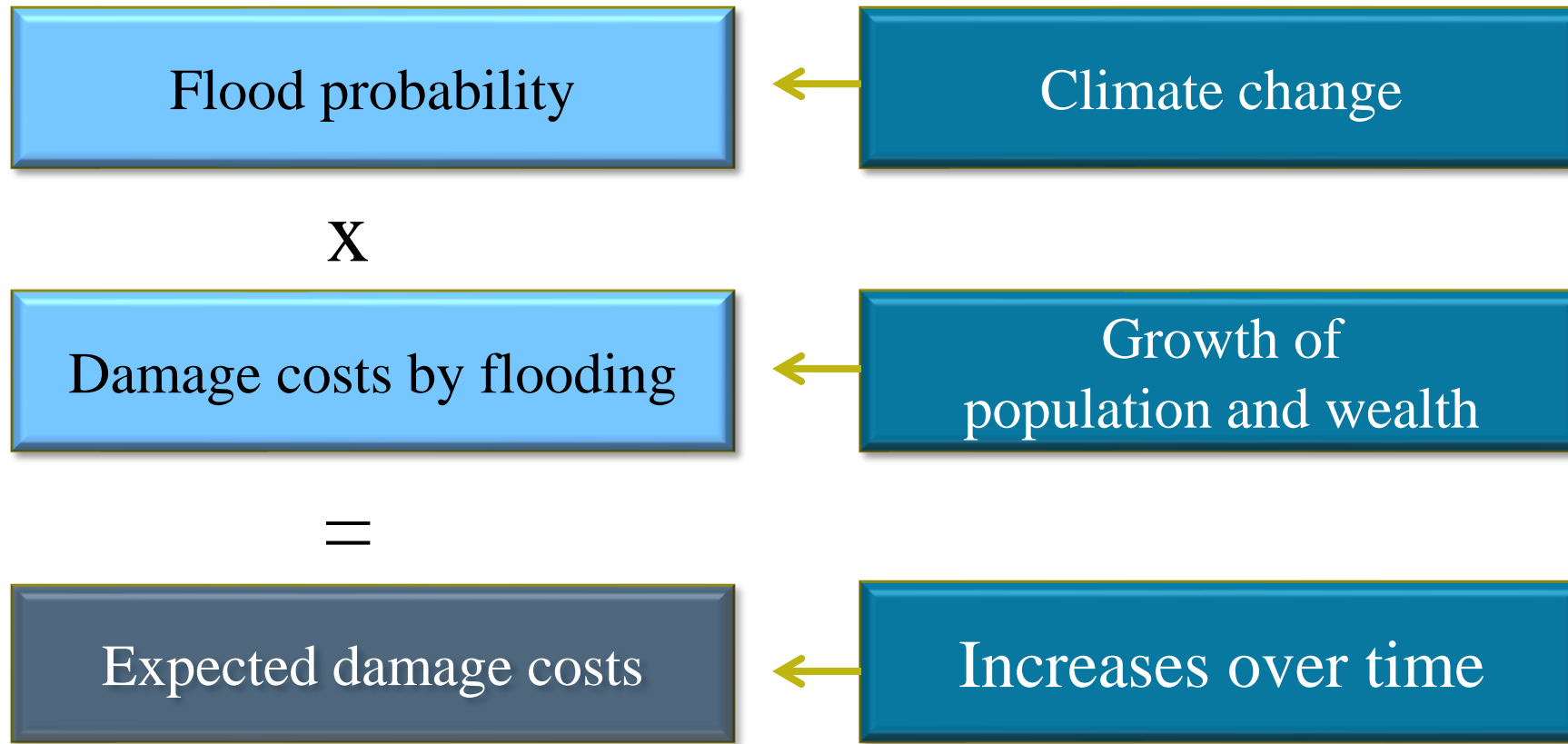


How much to heighten?



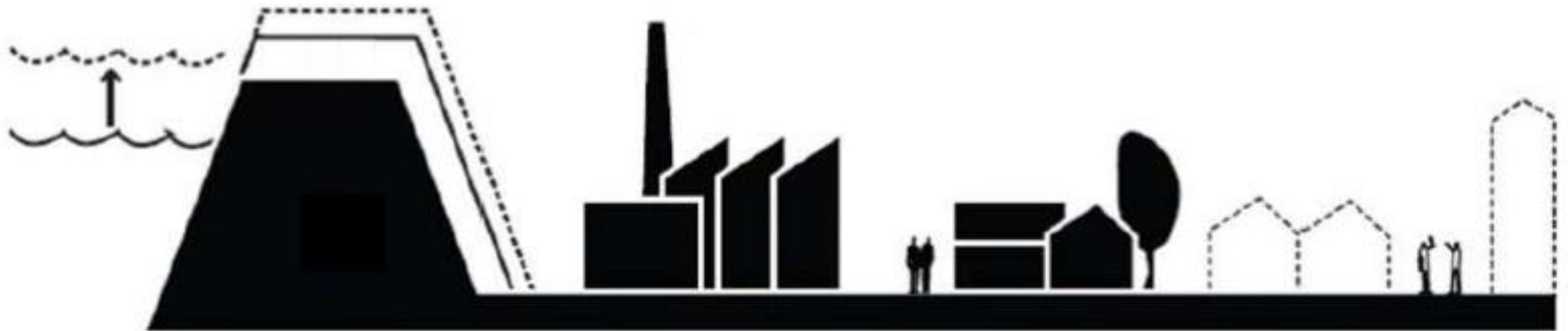


When to heighten?



Investment costs for heightening dikes

- Fixed costs involved
- Nonlinear in the heightening



Expected damage costs

- Flood probability x damage cost
- Nonlinear in the heightening





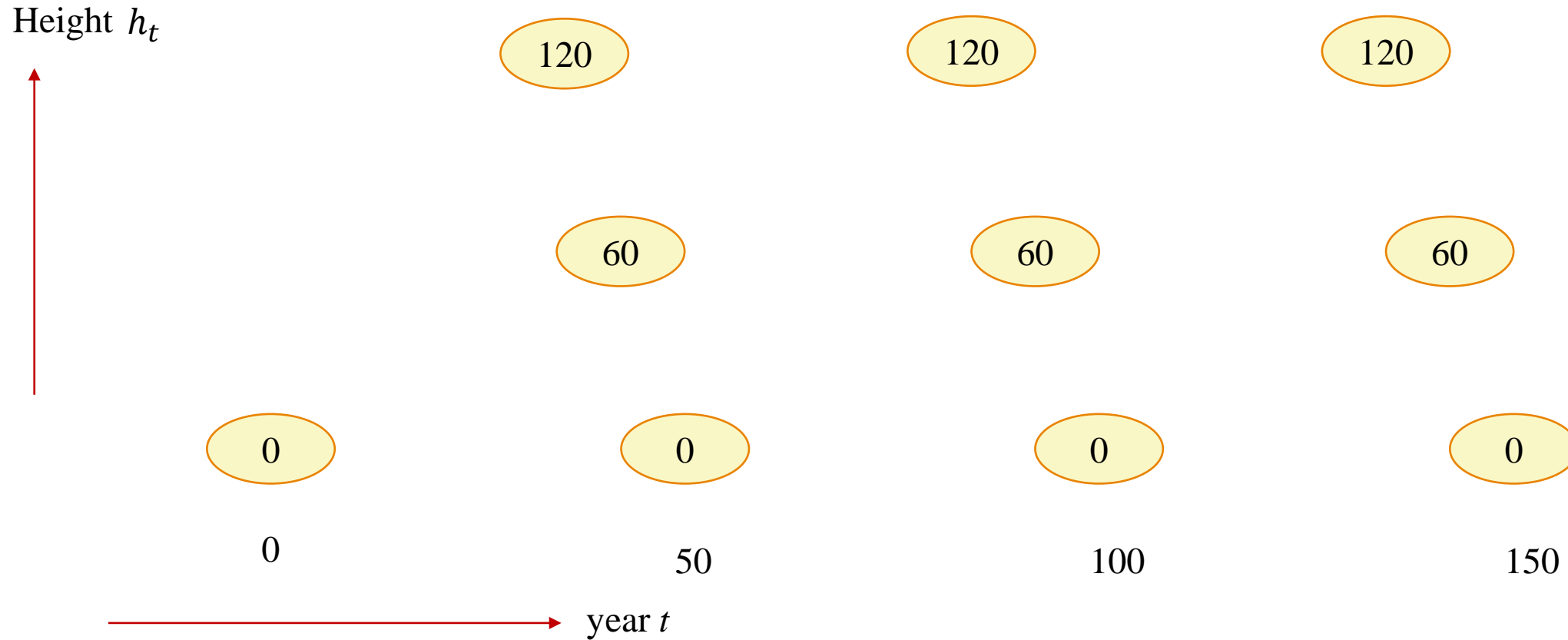
Suppose we have collected all these data ...

How to find the optimal height over time?

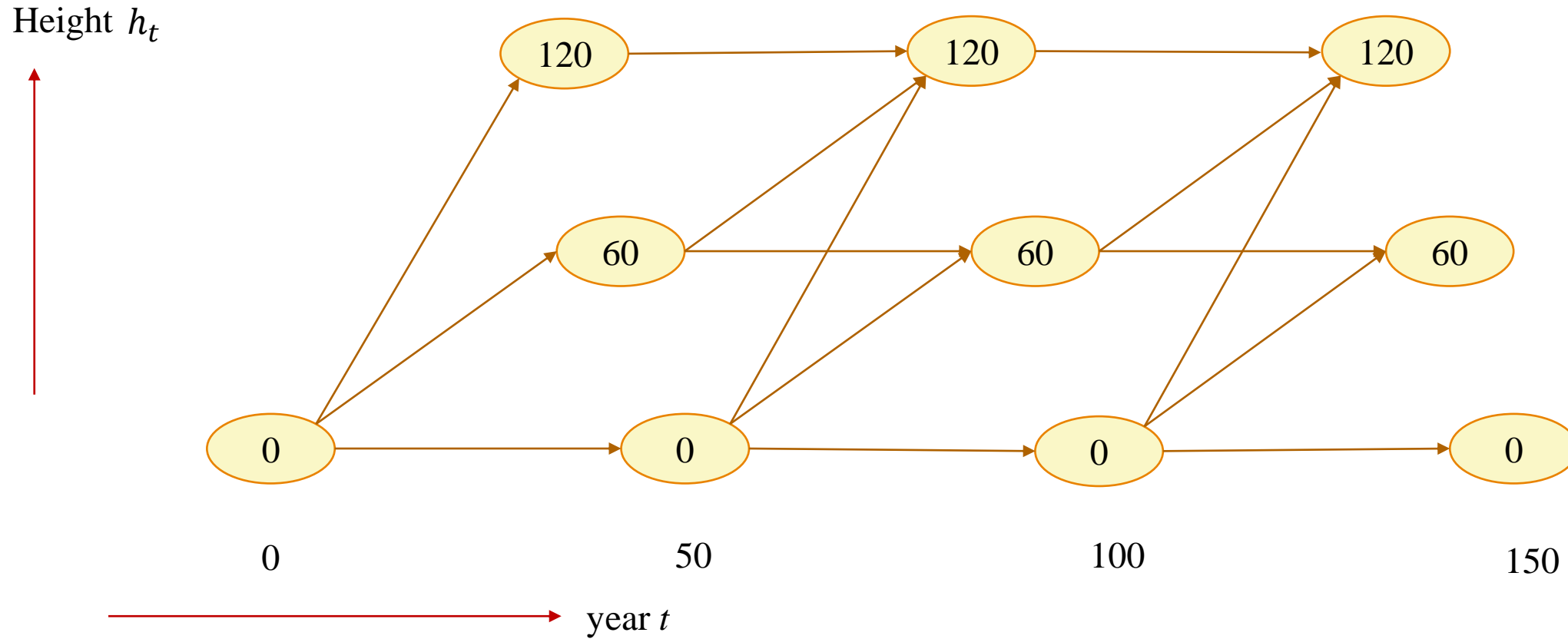


1. Shortest Path Model

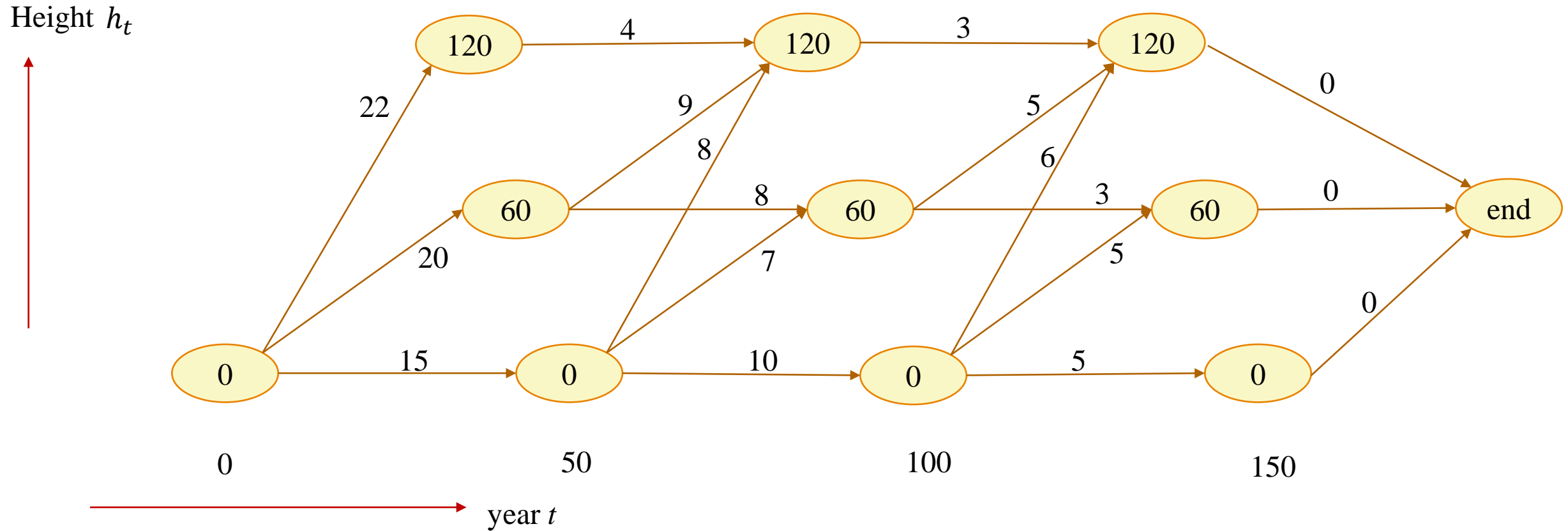
Graph representation (1)



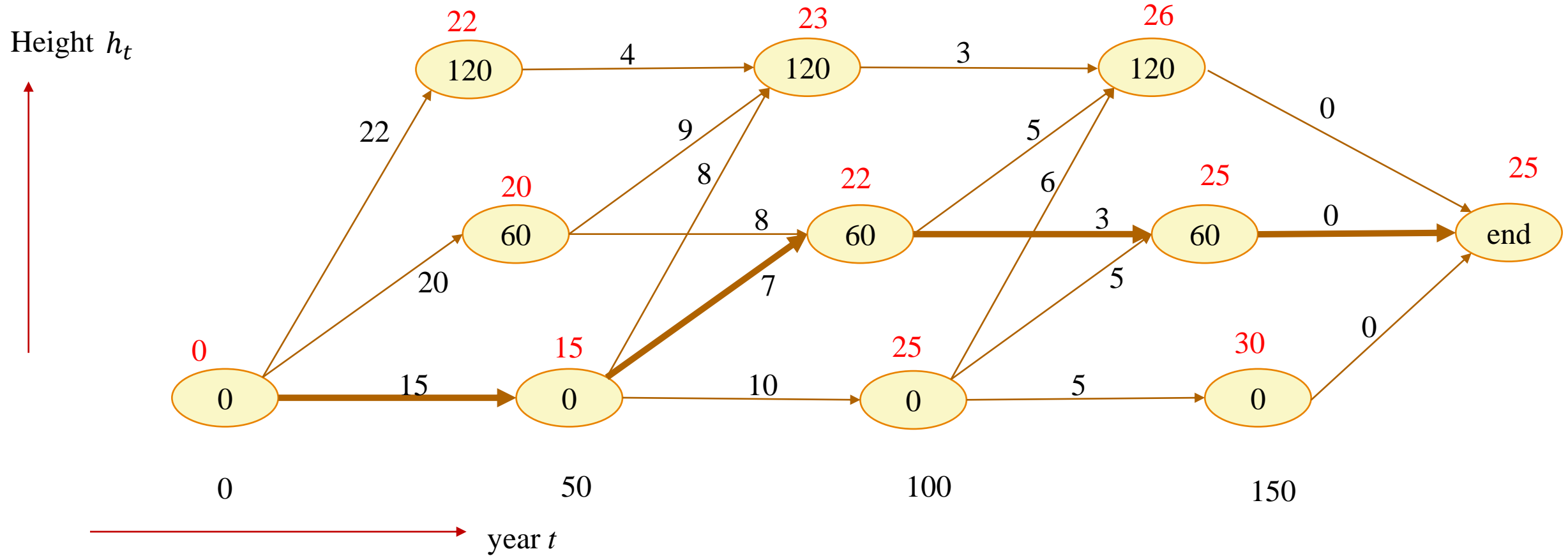
Graph representation (2)



Graph representation (3)



Shortest path – optimal strategy





Shortest path algorithm (Dijkstra)
solves the problem
within a few **seconds!**



Two reasons for using nonlinear optimization model

To get insight in structure of solution

State space is too huge for so-called nonhomogeneous dike ring areas





2. Nonlinear Optimization Model

--- **homogeneous case** ---



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Optimal Strategies for Flood Prevention

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[Online Appendix artikelMS_2015_11_17.dvi](#)



Formula for flood probability

Flood probability in year t :

$$P_t = P_0 e^{\alpha(\eta t - h_t)},$$

α : parameter of exponential distribution

η : structural increase in water level per year

h_t : cumulative heightening up to year t



Formula for damage

The potential damage by flooding in year t is defined as :

$$V_t = V_0 e^{\gamma t} e^{\zeta h_t},$$

where

γ : economic growth rate in dike ring (per year)

ζ : increase of damage per cm of dike heightening

Formula for investment costs

$$I(h^-, u) = \begin{cases} 0 & \text{if } u = 0, \\ (c + bu)e^{\lambda(h^- + u)} & \text{if } u > 0, \end{cases}$$

where the constants c , b , and λ are such that $c > 0$, $b \geq 0$, and $\lambda \geq 0$, with $b + \lambda > 0$, and h^- is the height of the dike just before a heightening by u .

Nonlinear optimization model

Extension of: Van Dantzig (Econometrica, 1956)

$$\min \left\{ \frac{S_0^-}{\beta - \delta_1} \sum_{i=0}^{\infty} e^{-\theta \sum_{\ell=1}^i u_{\ell}} [e^{(\beta - \delta_1)t_{i+1}} - e^{(\beta - \delta_1)t_i}] + \sum_{i=1}^{\infty} I \left(\sum_{\ell=1}^{i-1} u_{\ell}, u_i \right) e^{-\delta t_i} \right\}$$

Expected damage costs

Investment costs

Variables:

t_i is year of i -th heightening

u_l is amount of l -th heightening

$$I(h^-, u) = \begin{cases} 0 & \text{if } u = 0, \\ (c + bu)e^{\lambda(h^- + u)} & \text{if } u > 0, \end{cases}$$

$$\theta = \alpha - \zeta$$

$$\beta = \alpha\eta + \gamma$$

$$S_0^- = P_0 V_0$$

δ and δ_1 are discount rates

Periodic solution

- Problem is nonconvex
- Explicit formulas for optimal solution can be derived
- Optimality proof needs 24 pages
- Optimal solution is periodic

No.	Year t_1	t_2	u_1	p	v	Costs
10	46	103	57	57	57	40.0
11	42	101	62	59	62	110.2
15	0	51	56	52	53	545.2
16	4	58	53	54	53	1,089.7
22	13	75	54	62	54	309.3
23	56	104	55	48	55	20.1
24	8	51	62	43	62	297.3
35	0	41	64	42	60	345.2
38	0	51	65	51	62	172.1
41	0	60	101	63	75	325.9
42	15	77	72	61	72	79.2
43	8	73	73	65	73	1,304.8
44	0	50	77	55	50	206.5
45	0	46	62	51	41	33.7
47	0	51	65	52	56	64.1
48	0	41	58	42	51	403.0
49	32	85	46	53	46	74.0
50	0	59	96	59	62	53.5
51	35	87	40	51	40	54.2
52	4	61	46	58	46	245.4
53	0	67	87	69	66	307.5



3. Nonlinear Optimization Model

--- **nonhomogeneous case** ---



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Safe Dike Heights at Minimal Costs: The Nonhomogeneous Case

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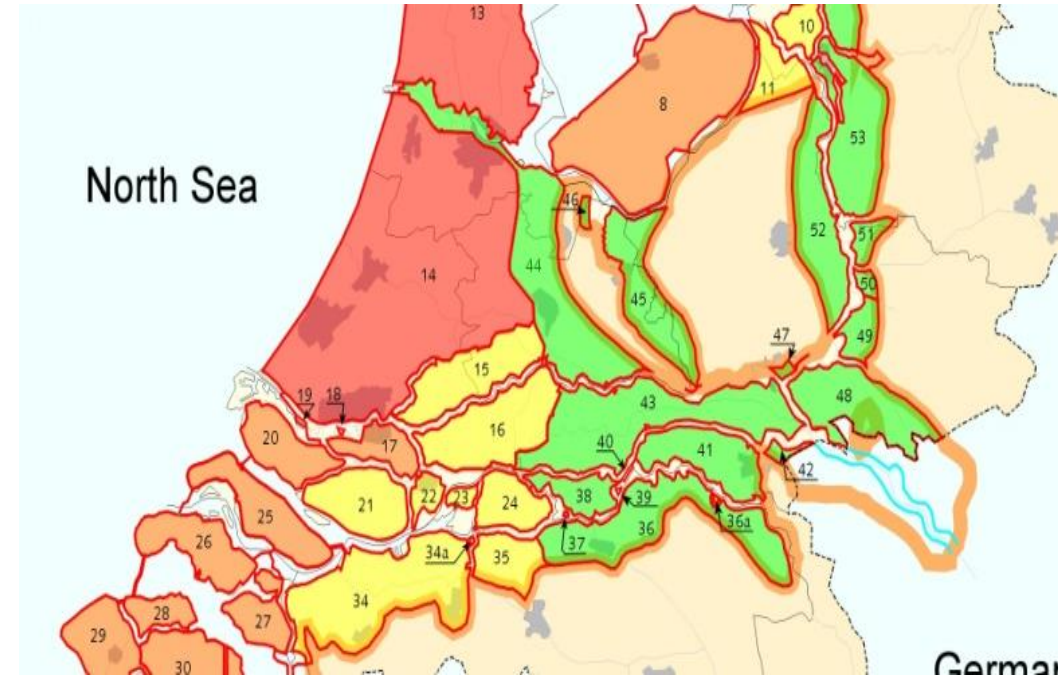
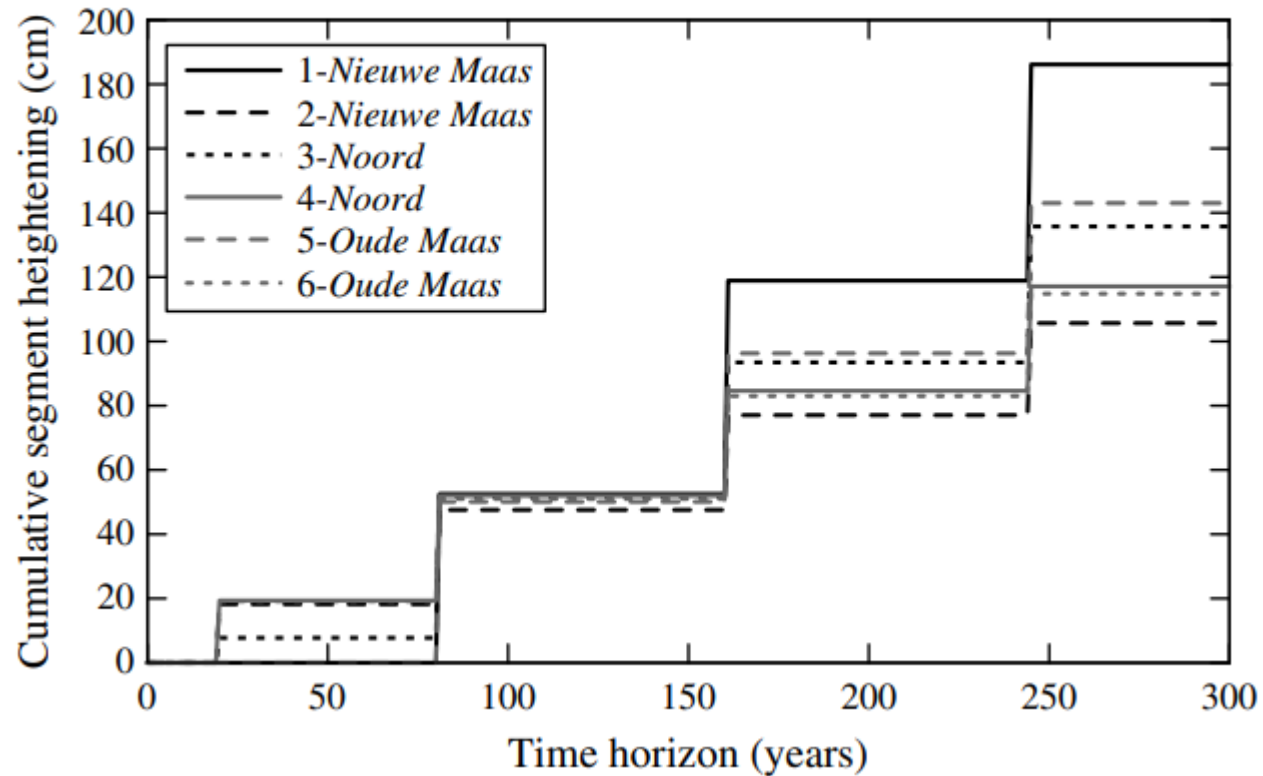
Mixed integer nonlinear optimization model

- nonconvex
- 2,400 continuous and 600 binary variables
- 3,000 constraints
- AIMMS, CPLEX, AOA
- robust version

$$\begin{aligned} \min \quad & \sum_{l=1}^L \sum_{k=0}^K \exp(-\delta t_k) (c_l y_{lk} + b_l u_{lk}) \exp\left(-\lambda_l \sum_{i=0}^k u_{li}\right) \\ & + \sum_{k=0}^K E_k + R \\ \text{s.t.} \quad & E_k \geq \frac{S_{l0}^-}{\beta_{1l}} \exp(\zeta(H_{l^*k} - H_{l0}^-) - \alpha_l h_{lk}) \\ & \cdot [\exp(\beta_{1l} t_{k+1}) - \exp(\beta_{1l} t_k)], \\ & \quad \quad \quad l = 1, \dots, L, k = 0, \dots, K, \\ & R \geq \frac{S_{l0}^-}{\delta} \exp(\beta_{1l} T - \alpha_l h_{lK} + \zeta(H_{l^*K} - H_{l0}^-)), \\ & \quad \quad \quad l = 1, \dots, L, \\ & h_{lk} = \sum_{i=0}^k u_{li}, \quad l = 1, \dots, L, k = 0, \dots, K, \\ & H_{lk} = H_{l0}^- + h_{lk}, \quad l = 1, \dots, L, k = 0, \dots, K, \\ & 0 \leq u_{lk} \leq y_{lk} M, \quad y_{lk} \in \{0, 1\}, \\ & \quad \quad \quad l = 1, \dots, L, k = 0, \dots, K, \\ & h_{lk}, H_{lk}, E_k, R \in \mathbb{R}, \\ & \quad \quad \quad l = 1, \dots, L, k = 0, \dots, K. \end{aligned}$$

Result for dike ring 17

Cumulative segment heightening dike ring 17.

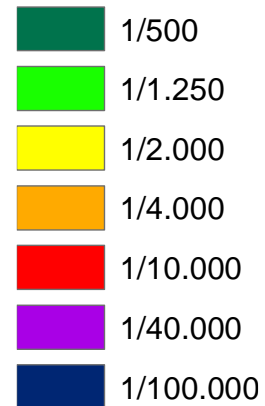
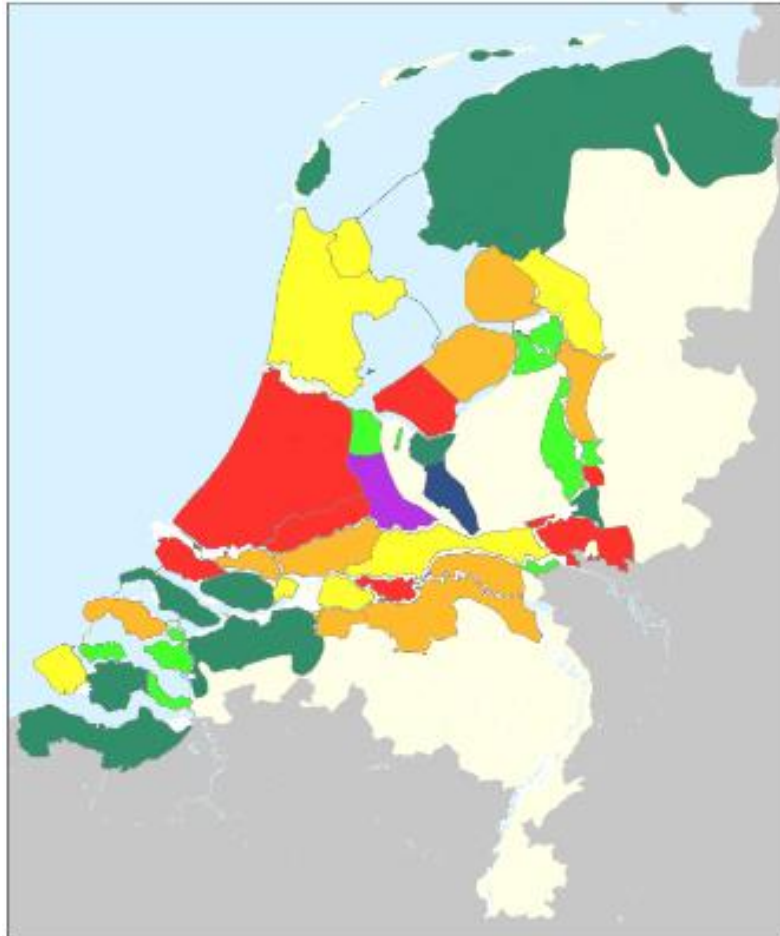




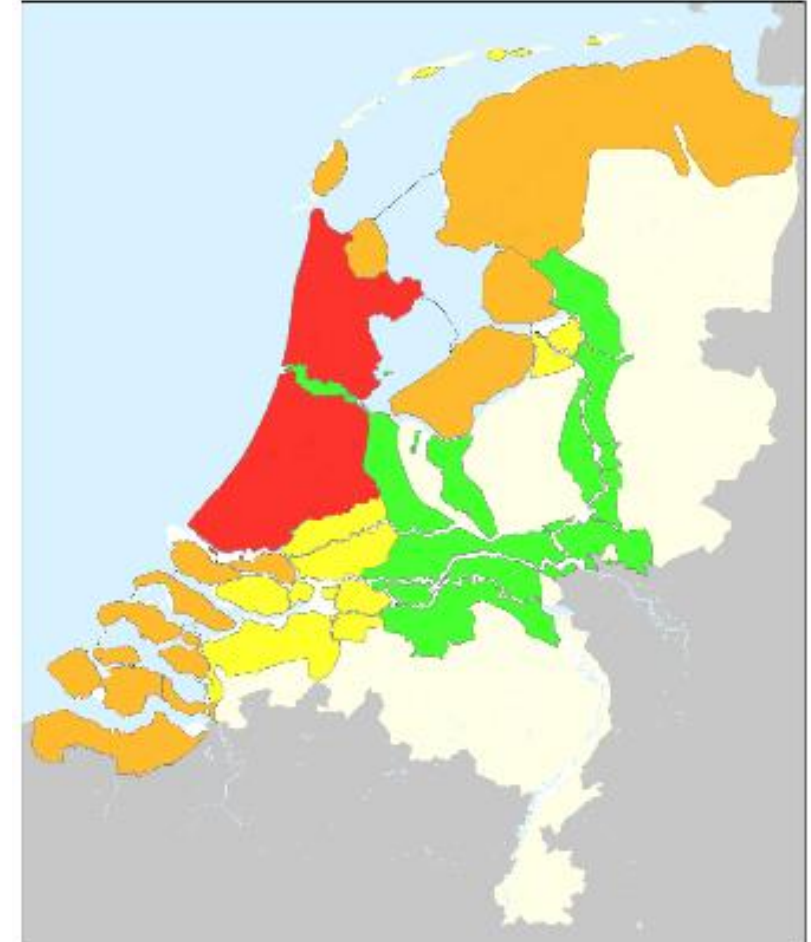
IMPACT

Impact of dike optimization project

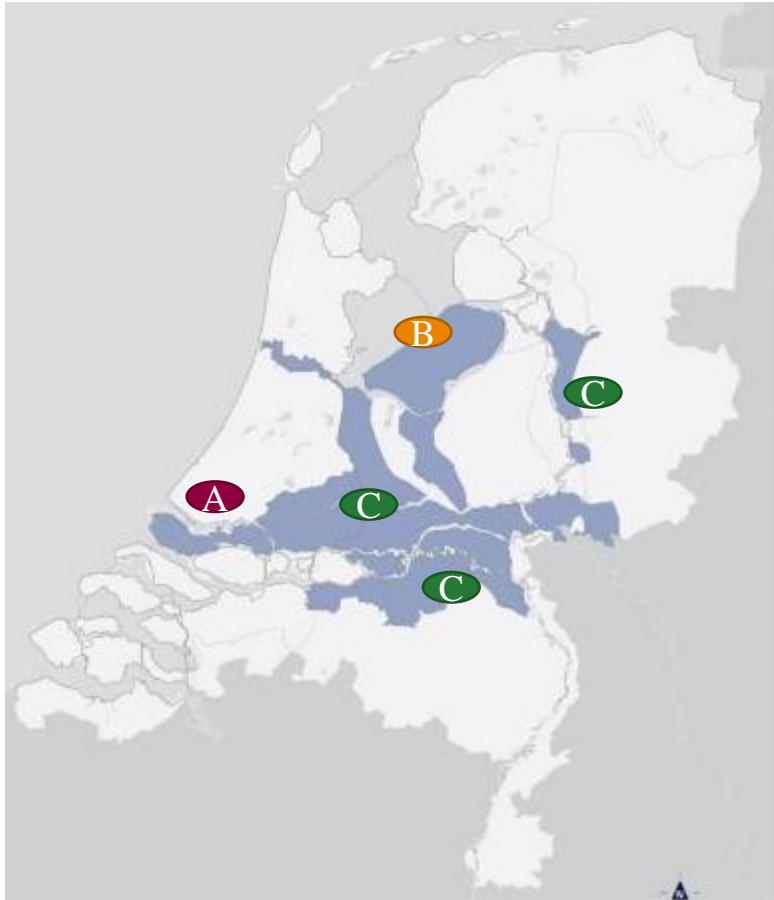
Efficient standards



Former standards



Final conclusion



Increase protection standards in:

- A** dike rings near Rotterdam
- B** dike ring Almere
- C** dike ring along Rhine and Meuse



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Ons kenmerk

Congratulations letter Minister Schultz van Haegen

Datum 9 april 2013
Betreft Franz Edelman Award

Geachte heer Kwadijk, beste Jaap,

Van harte gefeliciteerd met het winnen van de Franz Edelman Award voor jullie project '*Economically Efficient Standards to Protect the Netherlands against Flooding*'. Een mooie prestatie!

Zoals de jury vereiste, heeft jullie onderzoek grote impact op de samenleving: jullie rekenmethode heeft geleid tot nieuwe inzichten, betere veiligheidsnormen en flinke kostenbesparingen. Daarmee leveren jullie de basis voor het waterveiligheidsbeleid van de toekomst. Dat is iets om trots op te zijn!

Na TNT Express en de Nederlandse Spoorwegen zijn jullie bovendien de derde Nederlandse winnaars van Franz Edelman Award. Jullie bewijzen dus dat ons land op het gebied van innovatie en het optimaliseren van processen een naam hoog te houden heeft. En ook dat is iets om trots op te zijn.

Hartelijk dank voor jullie werk en nogmaals van harte gefeliciteerd met deze prijs.

Met vriendelijke groet,

DE MINISTER VAN INFRASTRUCTUUR EN MILIEU,

Melanie Schultz van Haegen



Implementation

Accepted by House of Parliament

Stated in law (January, 2017)

Implementation is still going on.





Every 30-40 years

New safety standards

(= Edelman award)

Every 6-12 years

Safety assessment

Every year

Maintenance





Questions ?